Lattice Boltzmann simulation of the flow of non-Newtonian fluids in porous media

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We present a LB study of the flow of single-phase non-Newtonian fluids, using a power law relationship between the effective viscosity and the local shear rate. Channel flow experiments were carried out to measure the velocity profiles. The simulation results are found to be in good agreement with theory. We also report simulations of the flow of non-Newtonian fluids in a 2-D porous medium.

1. Non-Newtonian flow

The flow behaviour of non-Newtonian fluids is of high interest to a broad range of disciplines in both science and technology. In particular, the oilfield industry is interested in fluids for downhole applications, such as hydraulic fracturing to increase the oil production from reservoirs with a low natural permeability. These are generally complex fluids (containing polymers, surfactants and/or colloids) which show non-Newtonian behaviour. Because of the nature of the application, the flow of such complex fluids in porous media is extremely important to the industry. In this paper, we describe a non-Newtonian lattice-Boltzmann (LB) model. The LB method is well-suited to study flow problems in complex geometries. Because real rocks are very complex, we have chosen to validate our non-Newtonian LB model for flow in simplified geometries; first we study flow in a 2D channel, followed by obstacle flow in a simple 2D porous medium. The Navier-Stokes equation for an incompressible fluid is:

$$\partial_t v_\alpha + v_\beta \partial_\beta v_\alpha + \frac{1}{\rho}\partial_\alpha p = \frac{1}{\rho}\partial_\beta \tau_{\alpha\beta} \tag{1}$$

Here, the fluid velocity is v_α, the density ρ, pressure p, deviatoric stress tensor $\tau_{\alpha\beta}$, and the Einstein summation convention is followed. The symmetric strain-rate tensor $e_{\alpha\beta}$ is defined as $e_{\alpha\beta} = \frac{1}{2}\left(\partial_\alpha v_\beta + \partial_\beta v_\alpha\right)$. For a Newtonian fluid, the stress tensor $\tau_{\alpha\beta}$ is simply given by $\tau_{\alpha\beta} = 2\mu e_{\alpha\beta}$. μ is the shear viscosity, independent of the shear rate. For a non-Newtonian fluid, the relation between the stress and strain tensors is not as simple; this is often represented by a shear-dependent viscosity $\mu = \mu(e_{\alpha\beta})$. A commonly used model for a non-Newtonian fluid is the power-law, or Ostwald-de Waele model, where the effective viscosity varies as the local shear rate $\dot{e} = \sqrt{e_{\alpha\beta}e_{\alpha\beta}}$, raised to the power of $(n-1)$, where n is the power law index:

$$\mu = \mu_0 \left|\dot{e}\right|^{(n-1)} = \mu_0 \left|e_{\alpha\beta}e_{\alpha\beta}\right|^{(n-1)/2} \tag{2}$$

A fluid with $n = 1$ is Newtonian. For a fluid with $n > 1$, the effective viscosity increases with shear, and the fluid is termed *dilatant*, or *shear-thickening*. For a fluid with $n < 1$, the effective viscosity decreases with shear, and the fluid is termed *pseudoplastic*, or *shear-thinning*.

1.1. *Channel flow*

Consider a two-dimensional channel, with parallel non-slip walls at $x = -w/2$ and $x = w/2$, and fluid with velocity $(v_x, v_y) = (0, v)$ between the walls. At equilibrium and constant pressure gradient G, equation (1) reduces to the simple form $G = -\nu\partial_x^2 v$. In a Newtonian fluid, a solution is the well-known Poiseuille velocity profile $v(x) = \frac{G}{2\nu}\left(w^2 - x^2\right)$, where x is the distance from the centre of the channel. For a non-Newtonian fluid, it can be shown[1] that a solution for the power-law fluid with mean velocity v_m is:

$$\frac{v(x)}{v_m} = \frac{2n+1}{n+1}\left[1 - \left(\frac{2|x|}{w}\right)^{(1+1/n)}\right] \tag{3}$$

1.2. *Darcy's Law*

Darcy's law[2] relates the flux q of a fluid through a porous medium to the pressure gradient G driving the flow. For a Newtonian fluid, $q = CG$. The coefficient C depends on both the fluid and the medium. It can be shown[1] that a generalization to power-law fluids is $q = CG^{1/n}$.

2. LB Simulations

Lattice Boltzmann models describe the evolution of a single-particle distribution function f_i defined on a discrete lattice of points $\mathbf{x}$, where each point is connected to its ith nearest neighbour by a vector $\mathbf{c}_i$. $f_i(\mathbf{x}, t)$ is defined so that the number density ρ and velocity $\mathbf{v}$ of the fluid are given by $\rho(\mathbf{x}) = \sum_i f_i(\mathbf{x})$ and $\rho\mathbf{v}(\mathbf{x}) = \sum_i f_i(\mathbf{x})\mathbf{c}_i$ respectively. In a lattice BGK model [3], the distribution function is updated by an advection step, with particles streaming along their velocity vectors to adjacent lattice sites, followed by a collision step, where the particles at each individual lattice site exchange momentum in such a way that the total mass and momentum of all the particles in the system are conserved. The collision step consists of a linear relaxation towards an equilibrium distribution $f_i^{(\mathrm{eq})}$. The combined advection and collision steps can be described in the lattice BGK equation:

$$f_i(\mathbf{x} + \mathbf{c}_i, t+1) - f_i(\mathbf{x}, t) = -\frac{1}{\tau}\left[f_i(\mathbf{x}, t) - f_i^{(\mathrm{eq})}(\mathbf{x}, t)\right] \tag{4}$$

The equilibrium distribution is a function only of particle number density ρ, and particle velocity $\mathbf{v}$ at a site, and takes the form of a polynomial chosen to give isotropic hydrodynamics[3]. It can be shown through a Chapman-Enskog procedure[4] that this algorithm produces the Newtonian incompressible Navier-Stokes equation with kinematic viscosity $\nu = \frac{1}{3}\left(\tau - \frac{1}{2}\right)$.

2.1. *Lattice BGK method for non-Newtonian flow*

Aharonov and Rothman[5] proposed a lattice Boltzmann model for power-law fluids which used a matrix collision operator. At each step of the algorithm, the local shear rate was calculated, and the collision operator tailored to produce the corresponding local kinematic viscosity. Rakotomalala *et al*[6] described a similar technique for a lattice BGK model, in which the single relaxation time τ is varied as a function of local shear rate to give the correct local viscosity. We use the same method, imposing the relation:

$$\tau = \frac{1}{2} + \left(\tau_0 - \frac{1}{2}\right) |e_{\alpha\beta} e_{\alpha\beta}|^{(n-1)/2} \tag{5}$$

This produces the appropriate behaviour of the kinematic viscosity; for $n = 1$, it reduces to a normal lattice BGK model with $\tau = \tau_0$.

2.2. *Non-Newtonian channel flow*

Simulations were run for fluids with $n = 0.56$, $n = 1$, and $n = 3.0$, with a body force applied to the fluid as in previous simulations[7]; the steady-state velocity profiles after 100000 timesteps were then compared with the theoretical profiles according to Equation (3), showing agreement to within 2%. Profiles of the normalized velocity $v(x)/v_m$ are shown in Figure 2(a), together with the predicted values.

2.3. *Flow through a simple porous medium*

Aharonov and Rothman[5] simulated flow through a simple porous medium consisting of a network of channels built on a 64×64 lattice, and were able to show favourable agreement with Darcy's law. We repeated their simulations, but using the lattice BGK model. Simulations were run for a variety of power laws, including $n = 0.7$, $n = 1.0$, and $n = 3.0$, producing a flow pattern qualitatively similar to that from the non-BGK model, as can be seen in Figure (1). Moreover, agreement with the generalized Darcy law was found. The normalized fluid fluxes and forcing rates are plotted in Figure 2(b), along with the curves predicted by Darcy's law: agreement is within 3%.

3. Conclusions

We have shown that a simple extension of the single relaxation time lattice BGK model can be used to examine flow of power law fluids through porous media. The method produces the correct theoretical velocity profile for channel flow, and may be used to treat flow of non-Newtonian fluids through porous media. In a following paper, we will report results from applying this model to a digitized slice of Berea sandstone.

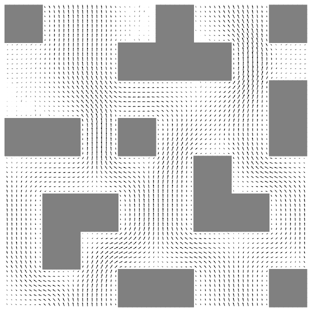
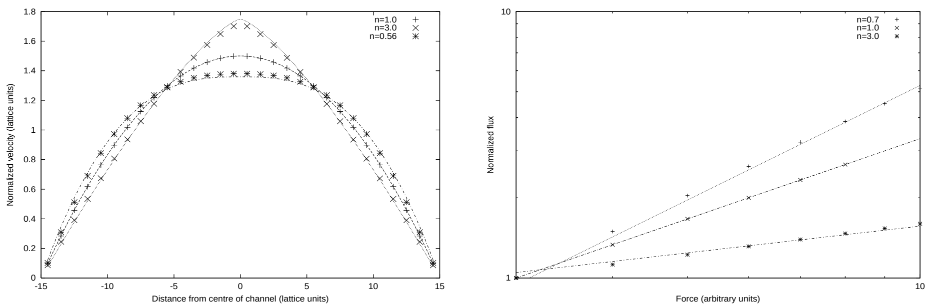


Fig. 1. Flow field through a simple porous medium for a shear-thinning fluid.



(a) Velocity profile for Newtonian and non-Newtonian channel flow

(b) Darcy law verification for a simple porous medium

Fig. 2.

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