Fixed Income
Quantitative Research

July 2005

# Inflation Derivatives Explained

## Markets, Products, and Pricing

Jeroen Kerkhof

- The inflation derivatives market has achieved critical mass, with an outstanding notional volume of over $100bn
- Inflation derivatives make it possible to isolate inflation risk from interest rate risk
- Zero-coupon inflation swaps dominate the market and form the building blocks for other inflation derivatives
- Real returns, breakeven inflation and seasonality are explained
- The mechanics, risks and uses of inflation derivatives are discussed
- ISDA 2005 inflation definitions are outlined

LEHMAN BROTHERS
*Where vision gets built.®*

# CONTENTS

**1. Introduction** 3

**2. The Markets** 5

2.1. Growth and size ... 5
2.2. Market breadth ... 6
2.3. Market participants ... 6
2.4. Products overview ... 8

**3. Inflation Indices** 9

3.1. Euro area ... 9
3.2. France ... 10
3.3. United Kingdom ... 11
3.4. United States ... 11
3.5. Other indices ... 12
3.6. Seasonality in inflation indices ... 12

**4. Inflation Basics and Concepts** 16

4.1. Inflation, nominal value and real value ... 16
4.2. Inflation-linked cashflows and real bonds ... 16
4.3. Indices, indexation lags and announcements ... 19
4.4. Breakeven inflation ... 21
4.5. Components of breakeven inflation rate ... 23

**5. Zero-coupon Inflation Swaps** 25

**6. Inflation Analysis Framework** 29

6.1. Constructing an inflation curve ... 29
6.2. Inflation and interest rate risk ... 34
6.3. Counterparty risk ... 36
6.4. Rounding risk ... 38
6.5. Seasonality risk ... 38
6.6. Institutional risk ... 40

**7. Inflation Swaps and Futures** 41

7.1. Revenue inflation swap ... 42
7.2. OTC inflation bond ... 43
7.3. Period-on-period inflation swaps ... 43
7.4. Inflation asset swaps ... 45
7.5. Inflation futures ... 49

**8. Inflation Volatility Products** 52

8.1. Inflation caps and floors ........ 52

8.2. Inflation swaptions ........ 53

8.3. LPI swaps ........ 56

8.4. Inflation spread options ........ 57

**9. Structured Inflation Products** 59

9.1. Total return swaps ........ 59

9.2. Inflation-linked equity ........ 61

9.3. Inflation-linked equity options ........ 61

9.4. Inflation-linked credit default swaps ........ 62

9.5. Inflation-linked CDO........ 64

**10. Legal, Regulatory, and Accounting Issues** 66

10.1. ISDA inflation derivatives documentation ........ 66

10.2. Regulation for pension funds........ 67

10.3. Accounting standards ........ 67

**Appendix** 69

A.1. Calculating inflation asset swap spreads ........ 69

A.2. Estimation of seasonal patterns ........ 70

A.3. Duration and convexity analysis........ 71

A.4. Valuation of forward starting zero-coupon swaps ........ 72

A.5. Valuation of inflation caps and floors........ 73

A.6. Valuation of spread options and inflation-linked equity options ........ 73

A.7. Valuation of inflation swaptions........ 74

**References** 75

**Glossary** 76

**Summary of Notation and Definitions** 79

## 1. INTRODUCTION[1]

**Jeroen Kerkhof**
+44 (0)20 7102 3981
jkerkhof@lehman.com

In recent years the market for inflation-linked derivative securities has experienced considerable growth. From almost non-existent in early 2001, it has grown to about €50bn notional traded through the broker market in 2004, double the notional traded through the broker market in 2003. Rapid growth is expected to continue for the coming years. So far the growth has mainly been driven by the European market, but recently interest in the US market has picked up as well.

*The purpose of inflation derivatives is the transfer of inflation risk*

The primary purpose of inflation derivatives is the transfer of inflation risk. For example, real estate companies may want to shed some of their natural exposure to inflation risk, while pension funds may want to cover their natural liabilities to this risk. In their simplest form, inflation derivatives provide an efficient way to transfer inflation risk. But their flexibility also allows them to replicate in derivative form the inflation risks embedded in other instruments such as standard cash instruments (that is, inflation-linked bonds). For example, as we will see later, an inflation swap can be theoretically replicated using a portfolio of a zero-coupon inflation-linked bond and a zero-coupon nominal bond.

*Inflation derivatives allow for tailor-made solutions*

As is the case for the nominal interest rate market, the advantage of inflation derivative contracts over inflation bonds is that derivatives can be tailored to fit particular client demand more precisely than bonds.

With the introduction of unfunded inflation-linked products, inflation derivatives have for the first time separated the issue of funding from inflation risk. This has made inflation markets more accessible to parties with high funding costs and made it cheaper to leverage inflation risk. For instance, hedge funds are increasingly involved in inflation markets.

*Our focus is on explaining the mechanics, risk and valuation of inflation derivatives*

Before any participant enters the inflation derivatives market, a solid understanding of the mechanics, risks and valuation of inflation derivatives is essential. The aim of this report is to provide this understanding. We hope that readers will gain the necessary comfort and understanding to take advantage of the new opportunities that the inflation derivatives market presents.

This report is structured as follows. Chapter 2 introduces the reader to the inflation market by presenting an overview of the market's growth, products and participants. It explains how the market developed and helps readers understand the likely future evolution of the inflation-linked bond and inflation derivatives markets.

Chapter 3 discusses the main indices used in both the inflation cash and derivatives market. Furthermore, it discusses seasonal patterns in inflation time series.

Chapter 4 discusses the basics of inflation markets. It explains key concepts such as real returns, real bonds, breakeven inflation and indexation lags.

Chapter 5 introduces the main inflation derivative security, the zero-coupon inflation swap. The zero-coupon inflation swap can be used as a building block for almost all other inflation derivatives.

Chapter 6 discusses the inflation risk framework. The chapter shows how to construct an inflation curve from zero-coupon inflation swaps incorporating seasonality effects. It also looks at inflation, interest rates, counterparty, rounding, seasonality and institutional risk.

Chapter 7 discusses the plain vanilla inflation derivatives products. The most common are inflation swaps, in their various forms, and asset swaps on inflation-linked bonds – a key link between cash and derivative securities. Investors who participate in the inflation

[1] *The author thanks Albert Brondolo, Robert Campbell, Giada Giani, Borut Miklavcic, Alvaro Mucida, Dominic O'Kane, Yan Phoa, Rami Shakarchi, and Fei Zhou for valuable comments and discussions.*

cash market stand to lose a valuable source of information if they do not understand the derivatives market, and vice versa, as the two markets are intimately related. We also discuss the recent inflation futures market.

Chapter 8 considers inflation products with optionality, such as caps, floors and swaptions. We also discuss LPI swaps which have an option-related payoff.

Chapter 9 discusses more complex derivative structures, including some hybrid structures. This is relatively unexplored territory in the inflation world, and is still driven exclusively by client specific needs. However, interest in these more advanced derivative securities is growing and they are likely to be the next area of growth of the inflation market.

Finally, Chapter 10 discusses the standardisation and definitions of the products by ISDA. Furthermore, it considers the influence of regulation and accounting standards on the inflation market.

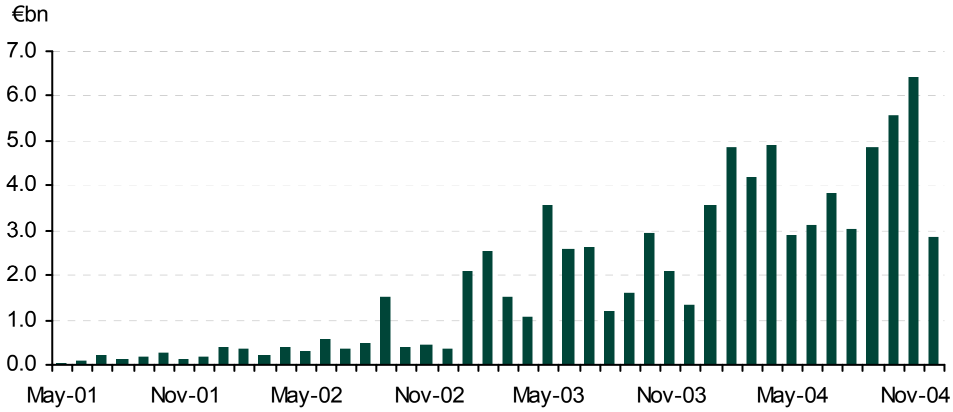
## 2. THE MARKETS

### 2.1. Growth and size

*Inflation swap volumes in the broker market have doubled in 2004*

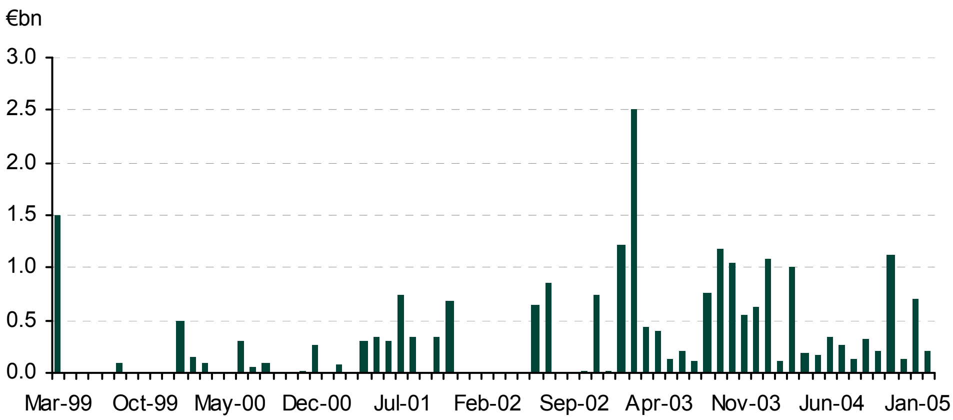
The inflation derivatives market has grown from an almost non-existent and fairly exotic branch of the interest rate market to a sizeable market with substantial growth potential. The pace of this growth has been swift. This is evidenced by inter-dealer data on notional volumes traded (Figure 2.1). Volume doubled in 2004 to about €50bn from about €25bn in 2003. Note that these numbers exclude swaps between banks and their clients, for which the volume is difficult to estimate. The rapid pace of this growth in the past few years is partly attributable to the favourable macroeconomic environment. Historically, low returns on traditional fixed income products and a reluctance on the part of investors to take on risk in the form of other assets led to a rapid growth in demand for structured products in 2003, as can be seen in Figure 2.2.

**Figure 2.1. Monthly volumes of inflation swaps traded in the broker market**




*Source: ICAP; Lehman Brothers. The graph also includes non-Euro swaps.*

**Figure 2.2. Gross issuance of structured inflation-linked notes (excluding sovereigns)**




*Source: Dealogic Bondware; Lehman Brothers.*

*Notionals in the inflation market are currently at about 2% of notionals in the interest rates market*

However, even though the market has been growing rapidly, it still represents only a small portion of the total rates market. Current volumes of inflation swaps amount to about 1-2% of nominal interest rate swap volumes. For the inflation derivatives market to grow further, it will need to find a balance between demand and supply of inflation.

This is clear by looking at the different markets. Five-year inflation swaps currently trade with a 2-3bp bid-offer spread in the European market as demand to receive inflation is strong and inflation can be sourced from the European inflation-linked bonds issued by France (OAT€i) and Italy (BTP€i). With these sovereigns increasing the percentage of their debt portfolio linked to inflation, it is likely that the inflation swap market will develop further. A large bond market does not necessarily imply a large inflation swap market. For example, the US inflation-linked bond market (TIPS) is currently the largest inflation-linked bond market, almost twice the size of the European inflation-linked bond market, yet 5-year US inflation swaps currently trade with bid-offer spreads more than double those in the European market. Given the large size of the US inflation-linked bond market and the increased activity and liquidity for US inflation derivatives in 2004, we expect the US inflation derivatives market to grow faster than other markets.

With the development of the broader inflation market, spurred by increased sovereign issuance (by the UK, euro-area countries, the US, Japan), investors and borrowers alike are becoming more aware of the benefits of inflation-linked securities. Considering the benefits of inflation-linked debt for issuers, we think it likely that issuers will continue to increase their inflation-linked debt portfolio relative to their nominal one. For example, the Italian Treasury has stated that in a steady state it is seeking to issue roughly 15% of its gross debt in the form of inflation-linked bonds. If the ratio of inflation-linked debt to nominal debt of the major sovereigns grows to about 10 to 15%, we would expect the inflation derivatives market to grow in a similar fashion. This could amount to a five- to ten-fold increase in inflation swap notionals from current volumes.

### 2.2. Market breadth

*The inflation derivatives markets mainly focus on the same indices as the inflation-linked bond market*

In terms of actively traded inflation indices, the inflation derivatives market focuses largely on the same indices as the inflation-linked government market. The main markets are the European market using the HICPxT index from Eurostat, the French market using the FRCPI index from INSEE, the UK market using the RPI index from National Statistics, and the US market using the US-CPI index from BLS (see Chapter 3 for a detailed discussion). Currently, the European derivatives market is by far the largest, but the other markets have reached a reasonable liquidity as well.

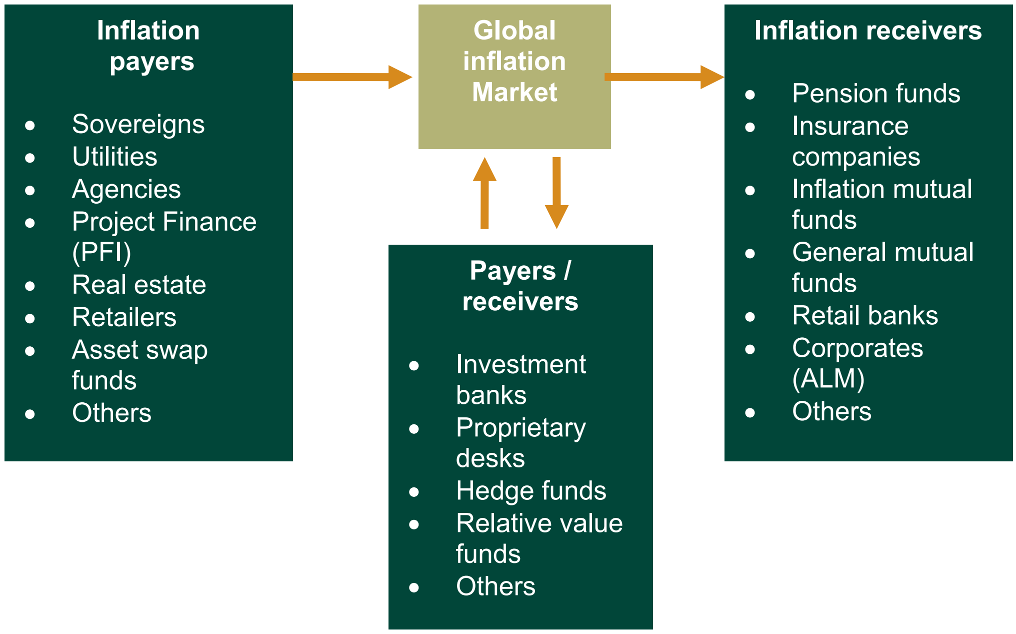
Clearly, the inflation derivatives market is not restricted to the indices for which (sovereign) issuers issue inflation-linked bonds. Indeed, the main strength of the inflation derivatives market is that it can provide anything the cash market can and more. For example, it is possible to structure inflation swaps linked to indices for which no inflation-linked debt is outstanding. However, bid-offer spreads will be quite sizable on these trades unless dealers can find a two-way market.

### 2.3. Market participants

*Sovereigns remain the main suppliers of inflation*

Inflation products attract a diverse group of investors such as banks, pension funds, mutual funds, insurance companies and hedge funds. Banks want to receive inflation on swaps to hedge inflation-linked retail products. Insurance companies and pension funds want to receive inflation to match their long-term inflation-linked liabilities. Figure 2.3 presents an overview of the market participants. We now separately discuss the natural payers and receivers and their incentives.

**Figure 2.3. Overview of participants in the global inflation market**




*Source: Lehman Brothers.*

### *2.3.1. Inflation payers*

Payers of inflation are entities that receive inflation cashflows in their natural line of business. Typical examples are sovereigns and utility companies. As their income is linked (either explicitly or implicitly) to inflation, they are ideally suited to pay inflation in the inflation market. Below, we list several reasons for issuing inflation-linked debt.

*Inflation-linked securities can improve the asset-liability mix*

1. For market participants with an income stream explicitly or implicitly linked to inflation, inflation-linked securities are natural hedge instruments against variations in inflation, making them attractive for asset-liability management purposes.

*By issuing inflation-linked debt, one can save the inflation risk premium*

2. Investors are interested in real returns rather than nominal returns. Inflation-linked bonds guarantee a real return, whereas nominal bonds guarantee a certain nominal, but uncertain real return. In order to compensate investors for taking the inflation risk accompanied with nominal bonds, nominal yields should be sufficiently high that the expected real return on nominal bonds is greater than the guaranteed real return on inflation-linked bonds. This additional yield on the nominal bonds is called the inflation risk premium. By issuing inflation-linked debt the issuer can thus save the inflation risk premium (or part of it).

3. In periods of shortage, inflation payers may prefer to pay low initial cashflows making up for this with higher payments later on. As inflation is typically positive, inflation-linked bonds fit such a cashflow structure.

*Diversification benefits for investors*

4. Another reason for issuing inflation-linked debt is to attract an investor base that is interested in the potential diversification benefits offered by these securities.

### *2.3.2. Inflation receivers*

Inflation receivers are typically entities that need to pay inflation-linked cashflows in their natural line of business. Examples include pension funds and insurance companies (e.g. via selling additional pension coverage policies). As their liabilities are linked (either explicitly or implicitly) to inflation, they are ideally suited to receive inflation in the inflation market. For instance, as pension funds need to minimise their shortfall risk[2] with as low as possible pension premiums, they should have a natural interest in

[2] *The risk that their assets drop below their liabilities.*

inflation-linked securities to match their inflation-linked liabilities. Having inflation-paying securities on their asset side can substantially reduce their shortfall risk as the value of their inflation-linked assets increases and decreases with their liabilities. The same holds for insurance companies selling inflation-linked additional pension coverage policies. Therefore, typically these institutions receive inflation in the inflation market.

Another important group of inflation receivers is the retail investment market. Although most people invest in inflation-linked cashflows via their pension schemes, many investors prefer to additionally invest directly in inflation-linked securities.

#### *2.3.3. Other players*

*Diversification and relative value opportunities make the inflation markets attractive for every investor*

Besides investors who are drawn to inflation markets in their natural line of business, inflation markets can offer attractive features for “inflation-neutral” investors. As inflation risk is not separately traded or is traded as a combination of other risks, it should provide diversification benefits for any investor. Furthermore, attractive relative value opportunities can arise for inflation-neutral investors.

### 2.4. Products overview

*Zero-coupon inflation swaps dominate the inflation derivatives market*

There are a number of instruments that can be classified as inflation derivatives, ranging from the standard zero-coupon inflation swap to structured inflation products. Currently the market is dominated by zero-coupon inflation swaps, but demand for more advanced products is growing as investors gain experience in the inflation market and become more aware of the benefits of these products.

*Zero-coupon inflation swaps are the building block of the inflation market*

Zero-coupon inflation swaps are the basic inflation products. They are detailed in Chapter 5. In Section 6.1, we will see that using zero-coupon inflation swaps, we can construct an inflation curve in a relatively straightforward manner. Besides zero-coupon swaps, year-on-year inflation swaps are other popular products. Year-on-year inflation swaps pay inflation over one year, for a period of several years (e.g. from March to March for a 5-year period). Period-on-period inflation swaps are discussed in detail in Section 7.1.

*Revenue inflation swaps can be used to hedge PFI*

Revenue inflation swaps pay the growth in the inflation index, that is, they pay the same real amount at all payment dates. In the UK, revenue inflation swaps are often used in order to hedge private finance initiatives (PFI). Instead of accessing capital markets directly (for instance, by issuing an inflation-linked bond), PFI project managers borrow from banks on a floating rate basis. As their revenues are often inflation-linked, they need to hedge their inflation risk which can be done using a revenue inflation swap. The structures are often more involved and more specific swaps need to be constructed, but they can typically be handled with an appropriately constructed portfolio of inflation swaps. Revenue inflation swaps are treated in detail in Section 7.1.

*Demand for inflation options is growing*

Demand for option structures is increasing as well. Caps and floors are useful instruments to generate partial indexation schemes. For example, the 1995 UK Pensions Act requires schemes to index against the RPI, but with an annual growth cap of 5% and annual growth floor of 0%. LPI swaps are instruments directly linked to such a liability stream. Other popular products are inflation swaptions. Inflation swaptions give companies with known future inflation-linked cashflows the opportunity to enter into an inflation swap when they start having inflation-linked exposures. Furthermore, they are used to create callable and cancellable inflation swaps. Inflation options are discussed in Chapter 8.

*Inflation hybrids open a new range of possibilities*

Another interesting trend is to include inflation-linked cashflows in payoff schemes of other asset classes, such as equity and credit. These so-called inflation hybrid products have been popular with retail customers. For instance, around 2001 most of the retail products sold in Italy were inflation and capital-protected equity-linked notes. Many varieties of inflation-linked hybrid structures are possible. We treat some of these in Chapter 9.

## 3. INFLATION INDICES

Any inflation-linked product needs a reference measure of inflation. These are known as inflation indices. In this section we describe the inflation indices used in the most important inflation markets.

### 3.1. Euro area

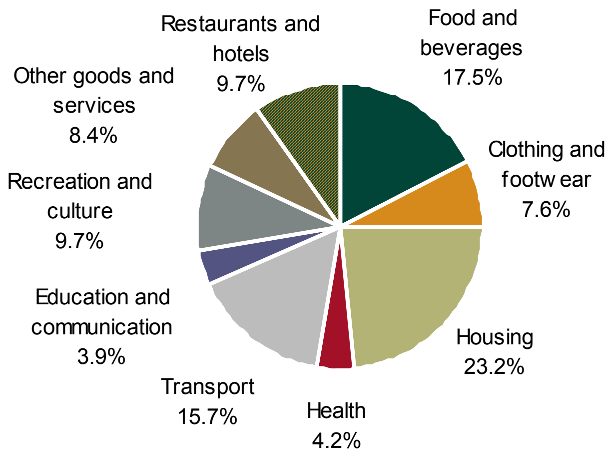
*HICP excluding tobacco is the main inflation index*

The euro area inflation swap market is currently by far the most liquid, active and transparent inflation swap market. The benchmark index for the euro area is the non-seasonally adjusted Euro HICPxT (Harmonised Index of Consumer Prices excluding Tobacco) published by Eurostat.[3] Consumption levels of the 12 different countries are used to weight the index. As countries accede to the monetary union, they will be included in the index. It is typically published two weeks after the end of the month. For instance, the Euro HICPxT index value for March is announced on about 15 April. The index announced is called the unrevised index. Eurostat might revise the index if after gathering more data they feel their initial announcement was inaccurate.

*Unrevised indices are used for cashflow computations*

Although the value of the index can be revised, the unrevised version is used in both the cash and the derivatives market. Eurostat publishes the Euro HICPxT index to the OATEI01 page on Reuters and the CPTFEMU<Index> on Bloomberg. The major constituents of the index are given in Figure 3.1. The base year for HICPxT is 1996, meaning that the average index value of HICPxT equaled a 100 during 1996.

**Figure 3.1. Constituents of HICP excluding tobacco**



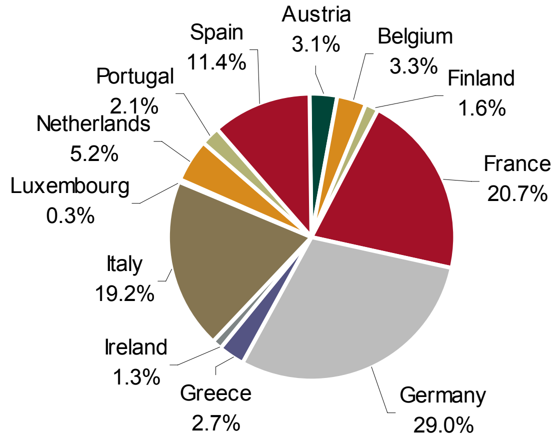

*Source: Eurostat; Lehman Brothers.*

*Country weights are based on GDP*

The Euro HICP excluding tobacco is a weighted sum of the euro-area countries' HICP indices. The country weights for the index are given in Figure 3.2 for 2005. The country weights are adapted on an annual basis based on GDP.

[3] See www.europa.eu.int/comm/eurostat/ for more information.

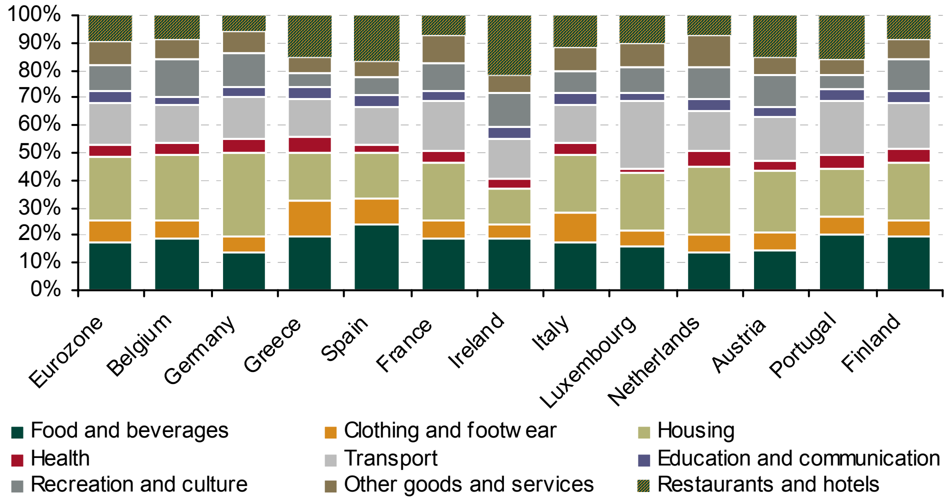
**Figure 3.2. Country weights of HICP excluding tobacco for 2005**




*Source: Eurostat; Lehman Brothers.*

The item weights of the Euro index will also vary as a consequence of varying country weights due to the fact that the individual HICP indices have varying item weights. Figure 3.3 shows the weights for the different categories for individual euro-area countries.

**Figure 3.3. Constituents of HICP ex-tobacco indices for individual countries**




*Source: Eurostat; Lehman Brothers.*

*The constituents of the inflation index vary heavily between countries*

The weights can vary substantially. For instance, housing accounts for about 30% of the German HICPxT while it accounts for only about 16.5% of the Spanish HICPxT. And restaurants and hotels make up about 17% of the Spanish HICPxT, but only about 5.7% of the German HICPxT.

## 3.2. France

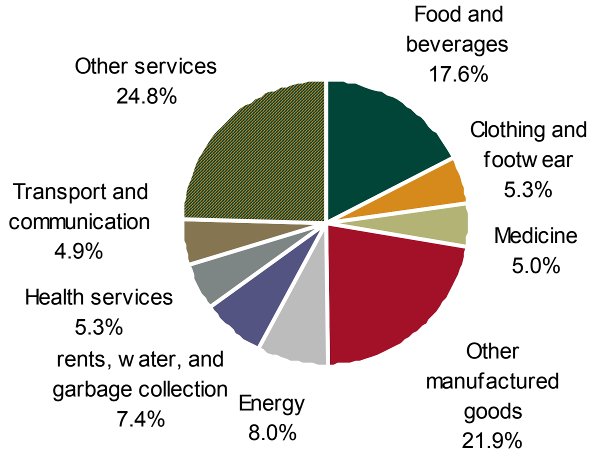
*French CPI excluding tobacco is the main index*

When France originally decided to issue inflation-linked debt, there was considerable debate about which index the issue should be linked to. A national index was likely to be a better match to the government's liabilities, while the Euro HICPxT index would appeal to international investors. Given the fact that the Euro HICPxT index was relatively new at the time of first issuance, the non-seasonally adjusted French CPI (Consumer Price Index) was chosen.[4] The index for each month is published by INSEE on about the 22nd of the subsequent month. Again the unrevised index is used. The

[4] See www.insee.fr/en/indicateur/indic_cons/indic_cons.asp for more information.

French CPI is published by INSEE on the OATINFLATION01 page on Reuters and FRCPXTOB<Index> on Bloomberg. The base year is 1998. In Figure 3.4 we present the constituents of the French CPI.

**Figure 3.4. Item weights of French CPI**



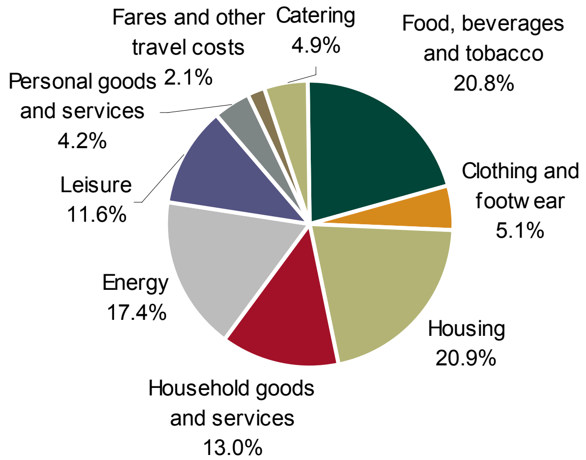

*Source: Insee; Lehman Brothers.*

### 3.3. United Kingdom

*RPI is the main index for inflation-linked securities*

In the UK market, inflation-linked securities are linked to the RPI (Retail Price Index).[5] This differs from the RPIX which excludes mortgage interest payments and until recently was the Monetary Policy Committee's (MPC) inflation target.[6] The unrevised version is used for inflation swaps. National statistics publishes the RPI index value for each month on about the 15th of the subsequent month. The Bloomberg ticker for RPI is UKRPI<Index>. The base reference equals January 1987. The major constituents of the RPI are given in Figure 3.5.

**Figure 3.5. Item weights of UK RPI**




*Source: National Statistic; Lehman Brothers.*

### 3.4. United States

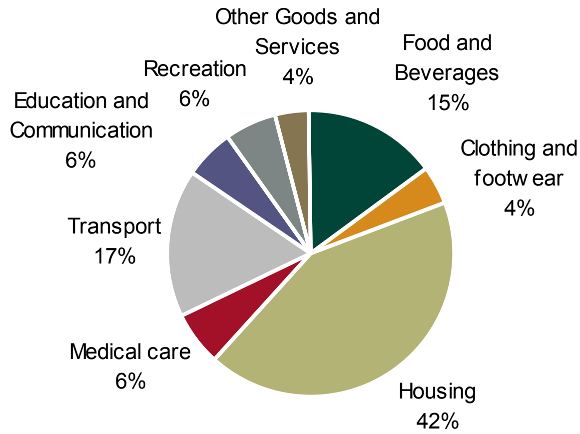
*The US CPI is the main inflation index*

Although the US inflation market has the largest backing of inflation-linked bond issues, liquidity remains substantially lower than in the euro-area, French and UK markets. It uses the same index as the TIPS market, the non-seasonally adjusted US City Average

[5] See www.statistics.gov.uk/CCI/nugget.asp?ID=22&Pos=2&ColRank=1&Rank=160 for more information.
[6] Since 2003 the MPC inflation target has been 2.0% on the UK CPI index.

All Items Consumer Price Index for all Urban Consumers (CPI-U) published by the Bureau of Labor Statistics (BLS).[7] The index can be found on Bloomberg, CPURNSA<Index>. The base is given by the average index of 1982-1984.[8] The constituents of the US CPI are presented in Figure 3.6.

**Figure 3.6. Constituents of the US CPI**




*Source: Bureau of Labor Statistics; Lehman Brothers.*

*Housing represents a big component of the US CPI*

Compared with the European indices, the US CPI has a very large housing component, and the value of the index is therefore largely driven by house prices and rents.

## 3.5. Other indices

*Italy is the most active derivatives market without national bond issuance*

By far the most active market without an underlying inflation-linked government bond market is Italy. The main index used is the Famiglie di Operai e Impiegati (FOI) excluding tobacco index. It is published by ISTAT and can be found on Bloomberg ITCPI <Index>. A small Spanish CPI market has developed without bond issuance support. The index used is the Spanish CPI, which is published by INE and available on Bloomberg SPCPI <Index>. Belgium could develop into a relatively significant market due to the fact that almost all inflation liabilities (e.g. rents) are linked to the same index, the health index. The index is published by the National Institute of Statistics and available on Bloomberg BECPHLTH <Index>. A small market has developed for Danish CPI due to a large Danish CPI issue to fund the transport link between Denmark and Sweden. The index used is Danish CPI published by Denmark Statistics and available on Bloomberg DNCPINEW <Index>. Other European indices have only rarely traded so far.

*In Japan a total return swap market has developed*

Outside of Europe, Japan launched the first inflation-linked government bond JGBi in March 2004. The inflation swap market has only developed slowly, but a market for total return swaps has been developed for JGBis. The index underlying the Japanese market is the Japanese CPI ex perishables index, which is published by the Ministry of Public Management, Home Affairs, Posts and Telecommunications. It is available on Bloomberg JCPNGENF <Index>.

## 3.6. Seasonality in inflation indices

*Inflation series contain strong seasonal effects*

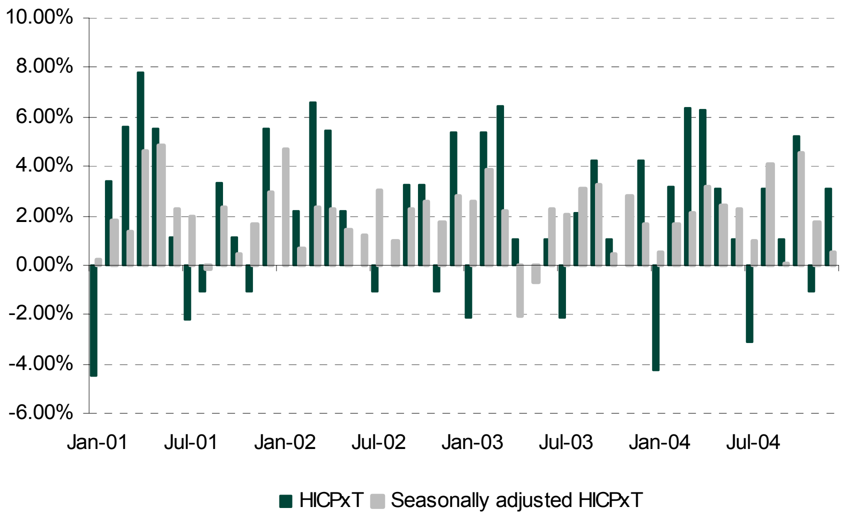
Figure 3.7 shows that the Euro inflation is quite volatile. However, part of the volatility can be attributed to seasonal effects. For instance, December inflation tends to be above the yearly average (the Christmas effect). Some of the main reasons for seasonality in inflation are varying food prices during the year and sales prices. Seasonal effects also

[7] *See www.bls.gov/cpi/home.htm for more information.*
[8] *The average US CPI value for Jan-1982 to and including Dec-1984 equals 100.*

differ significantly from one country to another, owing to different weights given to different items in the various national measures of inflation as seen in Figure 3.3. Moreover, there is a lack of harmonisation in the treatment of seasonal items. In general, national series are more volatile than euro aggregate because of different, often contrasting, national patterns.

A closer look at Figure 3.7 reveals that there is clearly higher inflation during the March-May period than the June-August period. We have estimated the seasonal pattern for the 2001-2004 period and plotted seasonally adjusted series based on these estimates besides the HICPxT inflation. This shows a much smoother pattern (the standard deviation was approximately half of the original series).

**Figure 3.7. HICPxT inflation history**




*Inflation history for the HICPxT index (revised index) for January 2001-December 2004.*
*Source: Lehman Brothers.*

In Figure 3.8 we have performed a regression analysis of inflation (see Appendix A.2 for an explanation of the seasonal dummy model estimation) for the entire HICPxT inflation history from February 1995 to December 2004. In order to investigate whether the seasonal patterns are statistically significant as well as being economically significant, we also report standard errors. We see in Table 3.8 that January, February, March, April, July, November and December are all statistically significant – different from 0 at a 5% confidence level.[9]

[9] *One can imagine more elaborate analysis considering, for example, mean-reversion effects, etc, but we will not pursue this route here (see, for example, Brockwell & Davis, 1991).*

**Figure 3.8. Seasonals for HICPxT**

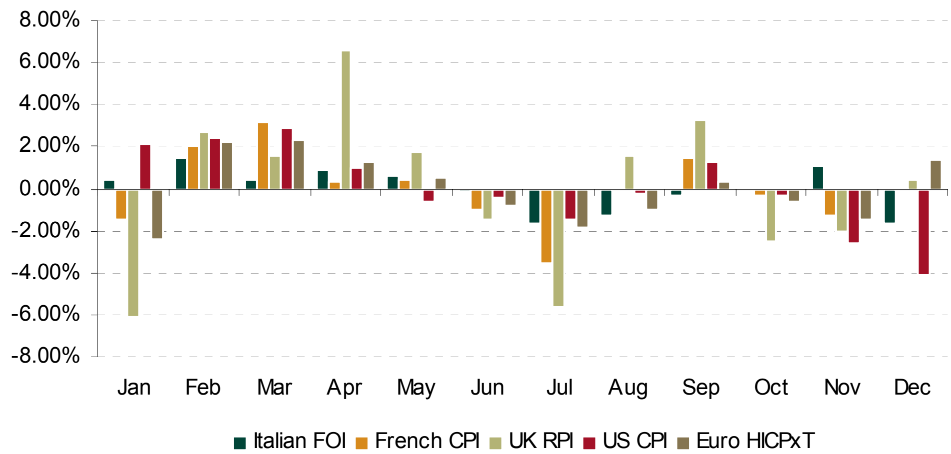
| Month | seasonal | std. errors | significant |
|---|---|---|---|
| January | -2.37% | 0.50% | Yes |
| February | 2.22% | 0.48% | Yes |
| March | 2.32% | 0.48% | Yes |
| April | 1.30% | 0.48% | Yes |
| May | 0.55% | 0.48% | No |
| June | -0.80% | 0.48% | No |
| July | -1.86% | 0.48% | Yes |
| August | -0.95% | 0.48% | No |
| September | 0.30% | 0.48% | No |
| October | -0.65% | 0.48% | No |
| November | -1.45% | 0.48% | Yes |
| December | 1.39% | 0.48% | Yes |

*This table shows the estimation results for HICPxT (revised series) seasonal patterns, std. errors and significance at a 95% confidence level. The data run from Feb-95 to Dec-04, giving us 119 observations, 9 for January and 10 for the other months.*
*Source: Lehman Brothers.*

*Different constituents and computation methods lead to different seasonal effects*

Figure 3.9 graphically shows the seasonal pattern present in the data. Figure 3.9 also shows the differences in seasonal patterns for different indices. The seasonality for all indices is mainly positive in the first months (apart from January) of the year and negative later on. It is interesting to see that Italy has a very low seasonal variance and the UK has a huge seasonal variance. For Italy fresh food prices are included as moving averages smoothing out part of the seasonality. The large positive April seasonal for the UK is due to taxation effects from the start of the fiscal year (the UK fiscal year runs from April to April). For Europe in general, the negative seasonals January and July are due to sales periods. The sales tend to be more aggressive in the UK resulting in very strong negative seasonals.

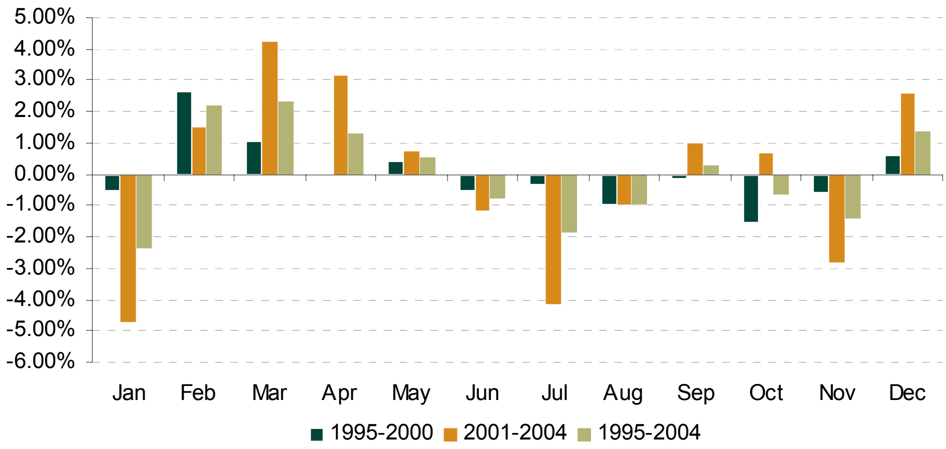
**Figure 3.9. Monthly seasonal components for different indices**




*Seasonal patterns are estimated using the seasonal dummy model on data from February 1995 to December 2004.*
*Source: Lehman Brothers.*

Seasonality estimates for the Euro HICPxT series are difficult to calculate as the index keeps evolving. For instance, since 2001 a number of countries added sales prices to their HICP numbers. Despite the limited available data, we see from Figure 3.10 that the seasonal pattern has become more pronounced recently.

**Figure 3.10. Monthly seasonal components HICPxT for 1995-2000 and 2001-2004**




*Seasonal patterns are estimated using the seasonal dummy model on inflation data from February 1995 to December 2000, January 2001 to December 2004, and the full sample.*
*Source: Lehman Brothers.*

Most inflation-linked products pay on an annual basis (e.g. OAT€i bonds and most inflation swaps), so the seasonal effects do not matter for initial pricing. However, seasonal effects are important for mark-to-market valuation. This mark-to-market valuation became increasingly important when the European Union adopted the accounting standards set by the International Accounting Standards Board (IASB) in 2005. These standards require defined benefit pension funds to reflect mark-to-market valuations of these schemes in their financial statements (see Section 10.3). In Section 6.1 we show how to build an inflation curve incorporating seasonal effects and explain how to mark-to-market inflation swaps.

## 4. INFLATION BASICS AND CONCEPTS

### 4.1. Inflation, nominal value and real value

*Investors care about real rather than nominal value*

Investors care about goods and services that money can buy, not money itself. For instance, people prefer a 5% increase in income and no increase in prices to a doubling of income and doubling of prices. This simple, but important and fundamental, economic axiom is crucial to the understanding of inflation-linked markets. Because the (consumer) market consists of a broad variety of products, a basket of goods and services is constructed that tries to represent the basket of goods and services used by a representative customer. For instance, in Figures 3.1, 3.4-3.6 we presented the major constituents of the Euro HICPxT, French CPI, UK RPI, and US CPI indices, respectively.

*An inflation index represents the relative value of a basket of goods and services*

We assume that all investors are and remain interested in this basket of goods and services. The nominal value of the basket of goods and services is computed at regular intervals (typically monthly). An inflation index is nothing more than the relative value of the basket. A base date is chosen at which the nominal value of the index is set to 100. If the nominal value of the basket equals €10,000 at the base date it means that the index will rise 1 point if the basket value increases by €100.

*Example 4.1*

Let us consider an investor with assets equal to €100,000. The investor can currently buy 10 baskets with his assets. A year later the index has risen from 100 to 102 (the cost of the basket has increased to €10,200). This means that inflation was equal to 2% (=102/100 -1). Besides the increase in the index, the nominal value of the assets of the investor has increased to €101,000. The nominal increase for the investor equalled:

$$\text{nominal change} = \frac{101{,}000 - 100{,}000}{100{,}000} = +1.00\%.$$

However, due to the inflation of 2% the investor can now only buy 9.90 (=101,000 / 10,200) baskets of goods and services. The real income change is therefore equal to:

$$\text{real change } = \frac{101{,}000/10{,}200 - 10}{10} = -0.98\%.$$

Of course, there is no need to compute this via the value of the reference basket. We find the same result using the nominal values and the inflation index:

$$\text{real change } = \frac{101{,}000/102}{100{,}000/100} - 1 = -0.98\%.$$

Even though the value of the investor's assets grew in nominal terms (+1%), in real terms his assets have decreased (-0.98%).

*Governments use inflation indices to provide real value certainty to its inhabitants*

One of the key tasks of most governments is to increase the real assets of its inhabitants. Most governments have legislation in place that gives employees the right to inflation compensation in their salary. This is typically achieved by a salary increase in wages depending on an inflation index at regular intervals (typically, annually). Also property rental prices are often linked to an inflation index, meaning that the property owners may increase rents according to the rise in the inflation index at pre-specified intervals.

### 4.2. Inflation-linked cashflows and real bonds

*Inflation-linked bonds can guarantee real returns*

As investors care about real income rather than nominal income, they prefer to invest in securities guaranteeing them a real return rather than a nominal one. In this section, we show how investors can get guaranteed real returns instead of nominal returns by using inflation-linked bonds in a world with inflation.

An inflation-linked zero-coupon bond is a bond that has a single payment at time $T$, its maturity date. We denote its value today (that is, time 0) as $D_{IL}(0,T)$. The nominal payment at maturity is equal to:

$$D_{IL}(T,T) = I(T),$$

the value of the index at maturity.[10] As investors are interested in real returns they value all cashflows relative to the index, $I$. Therefore, the inflation-linked bond has a real value equal to

$$I(T)/I(T) = 1$$

real unit at maturity. In order to get the real value of this inflation-linked bond today, $D_r(0,T)$, we need to divide the value of the inflation-linked bond by the current value of the inflation index, $I(0)$. The real value of an inflation-linked zero-coupon bond is given by[11]:

$$D_r(0,T) = \frac{D_{IL}(0,T)}{I(0)}.$$

We denote the current nominal value of €1 at time $T$ by $D_n(0,T)$, a nominal zero-coupon bond. Similarly, we denote the current real value of 1 real unit at time $T$ by $D_r(0,T)$, a real zero-coupon bond. Figure 4.1 illustrates the cashflows and values of an inflation-linked zero-coupon bond in both nominal and real terms.

**Figure 4.1. Inflation-linked payments in nominal and real terms**

| | **Today (*t=0*)** | | **Maturity (*T*)** |
|---|---|---|---|
| Nominal units | $I(0)D_r(0,T) = D_{IL}(0,T)$ | $\longrightarrow$ | $I(T)$ |
| Real units | $D_r(0,T)$ | $\longrightarrow$ | 1 |

The real return (that is, return in real units) on an inflation-linked zero-coupon bond is given by[12]:

$$y_r(0,T) = D_r(0,T)^{-1/T} - 1,$$

where $y_r(0,T)$ denotes the annualised real zero-yield and $D_r(0,T)$ denotes real value of an inflation-linked bond maturing at time $T$. Thus, using inflation-linked bonds one can get a guaranteed real return in the same way as nominal bonds allow one to get a guaranteed nominal return. The nominal return on an inflation-linked bond is uncertain and given by:

$$\left(\frac{I(T)}{I(0)D_r(0,T)}\right)^{1/T} - 1 = \left(\frac{I(T)}{I(0)}\right)^{1/T} D_r(0,T)^{-1/T} - 1,$$

[10] *In practice, a lag exists between the index date and the maturity date of the bond. We discuss inflation lags in detail in Section 4.3.*

[11] *In practice the value of the index is not known for each date, but it is only known at regular intervals (typically monthly). Furthermore, there is a delay between when the inflation takes place and when it is known. In Section 4.3, we explain how this problem is treated in practice and how we can assume that the value of the index can be observed for each date.*

[12] *This follows from the general definition that R(0,T) is defined as the annualised net return on security P for the period [0,T] using the following formula:*

$$\left(1 + R(0,T)\right)^T = \frac{P(T)}{P(0)},$$

*where P(0) denotes the current price and P(T) the price of the security at maturity, T. We assume no intermediate dividend payments in the above definition.*

*Example 4.2*

We assume that the current index equals 100, $I(0)$=100. The market trades an inflation-linked zero-coupon bond at 98.04% and a nominal bond at 96.15%, both with a time-to-maturity equal to 1 year ($T$=1). From these values we can get the annualised nominal yields and real yields in the following manner.

*Nominal yield on nominal zero-coupon bond*:

$$y_n(0,T) = D_n(0,T)^{-1/T} - 1 = \frac{1}{0.9615} - 1 = 4.00\%.$$

*Real yield on inflation-linked zero-coupon bond*:

$$y_r(0,T) = D_r(0,T)^{-1/T} - 1 = \frac{1}{0.9804} - 1 = 2.00\%.$$

An investor can thus lock-in a guaranteed nominal return of 4% or a guaranteed real return of 2%. Given the growth of the inflation index, we can calculate the real yield on a nominal bond and the nominal yield on an inflation-linked bond. Assuming the inflation index grows to 102, $I(T)$=102, these can be computed in the following manner.

*Real yield on nominal zero-coupon bond*:

$$\frac{1 \times I(0)/I(T)}{0.9615} - 1 = \frac{100/102}{0.9615} - 1 = 1.96\%.$$

*Nominal yield on inflation-linked zero-coupon bond*:

$$\left(\frac{I(T)}{I(0)D_r(0,T)}\right)^{1/T} - 1 = \left(\frac{102}{100}\right)\frac{1}{0.9804} - 1 = 4.04\%.$$

In Figure 4.2 we consider three scenarios for the index after a year: (1) the index remains the same, that is, no inflation; (2) the index grows to 102, that is, 2% inflation; the index grows to 104, that is, 4% inflation.

**Figure 4.2. Nominal and real returns on nominal and inflation-linked bonds**

| Inflation index | Nom. return nominal bond | Real return nominal bond | Nom. return IL bond | Real return IL bond |
|---|---|---|---|---|
| 100 | 4.00% | 4.00% | 2.00% | 2.00% |
| 102 | 4.00% | 1.96% | 4.04% | 2.00% |
| 104 | 4.00% | 0.00% | 6.08% | 2.00% |

*Source: Lehman Brothers.*

We see in Figure 4.2 that the nominal bond has a certain nominal return but uncertain real return, whereas the inflation-linked bond has a certain real return and an uncertain nominal return. Figure 4.2 also shows that for 2% inflation the nominal bond and the inflation-linked bond have almost identical nominal and real returns. In Section 4.4 we explain how one can compute the exact level of inflation for which the nominal and inflation-linked bonds have exactly the same returns.

As is the case in the nominal market, issuers do not issue zero-coupon bonds, but typically issue coupon bonds. In the same manner as for nominal bonds we can show that a coupon bond is nothing more than a portfolio of zero-coupon bonds with different maturities. Consider an inflation-linked coupon bond that pays a coupon equal to $c$ at times $T_1, \ldots, T_N$. We can write the nominal value of this bond at time $0 \leq T_1$ as follows:

$$\begin{aligned}B_{IL}(0,T_N) &= \sum_{i=1}^{N} cD_{IL}(0,T_i) + D_{IL}(0,T_N)\\ &= I(0)\left(\sum_{i=1}^{N} cD_r(0,T_i) + D_r(0,T_N)\right)\\ &= I(0)B_r(0,T_N).\end{aligned}$$

A detailed account of inflation-linked bonds is given in Brondolo (2005) and Deacon et al. (2004). Below we summarise the notation introduced in this section.

**Summary of notation used**

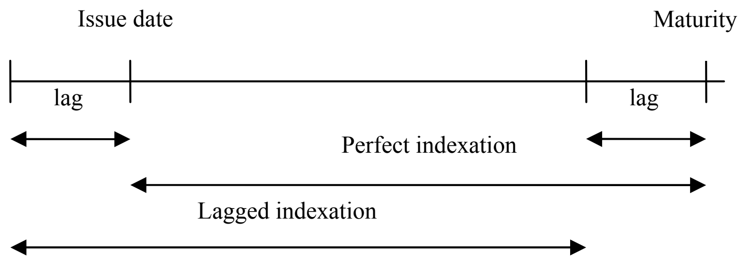
| | |
|---|---|
| $D_n(0,T)$ | Current value of nominal zero-coupon bond |
| $D_r(0,T)$ | Current value of a real zero-coupon bond |
| $D_{IL}(0,T)$ | Current nominal value of an inflation-linked bond |
| $y_n(0,T)$ | Yield on nominal zero-coupon bond |
| $y_r(0,T)$ | Yield on real zero-coupon bond |
| $I(T)$ | Value of index at time $T$ |

## 4.3. Indices, indexation lags and announcements

*Inflation lags lead to a lower degree of real value certainty*

As discussed before, the main purpose of inflation-linked securities is to provide real value certainty. In order to achieve a high degree of real value certainty the inflation-linked cashflows should be linked as closely as possible to contemporaneous inflation. However, in practice, the value of the index is not yet known for the cashflow date and a lagged index value is taken. As a result, investors have no inflation protection over the last period (typically, three months) of their inflation-protected security. They are compensated for this by receiving the inflation of the period preceding the purchase of the security. This is illustrated in Figure 4.3. In general, the inflation over the perfect indexation period is not equal to the inflation over the lagged inflation period leading to a lower degree of real value certainty.

**Figure 4.3. Indexation lag**




*This figure graphically illustrates the influence of lagged inflation on real value certainty.*
*Source: Lehman Brothers.*

Differences are likely to be bigger for longer indexation lags and more volatile inflation environments. Furthermore, the influence of the lag increases with decreasing time to maturity. Therefore, a small indexation lag is preferred for a high degree of real value certainty.

*Indexation lags are unavoidable*

There are two main reasons for indexation lags. First, it takes time to process consumer price data and compute inflation numbers. Due to the processing time, inflation is, typically, announced about two weeks after the month under consideration (for example, January inflation is announced on about 15 February). Second, a lag arises due to trading and settling of bonds between coupon payment dates. As for nominal bonds, inflation-linked bonds usually pay coupons; if the bond trades between coupon dates sellers should be compensated for having held the bond for part of the coupon period even though they will not receive the coupon. As for nominal bonds, this compensation is

*The UK used an eight-month lag to handle accrued interest*

effected via the payment of accrued interest. Two main methods of accrued interest payment are seen in practice. The oldest is the one employed by inflation-linked gilts in the UK market issued before 2005, where the next coupon is known at all times. This is achieved by using an eight-month lag consisting of a two-month period allowing for publication of the inflation index and six months for the accrued interest calculation (the inflation-linked gilts pay semi-annual coupons).[13]

*Canada introduced a new method for computing accrued interest*

*Reference numbers are computed for each day*

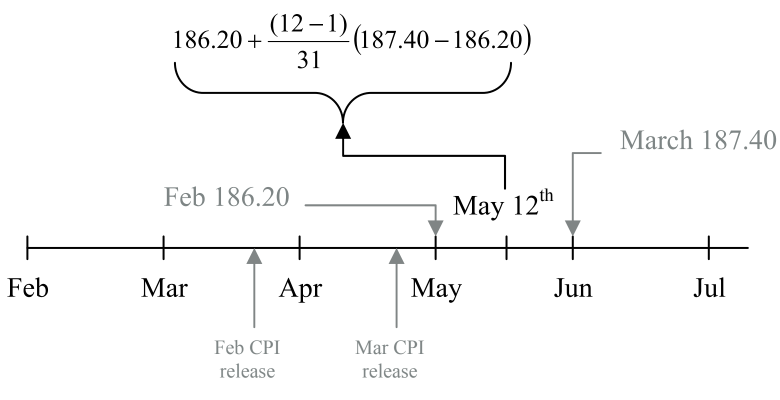
A more common and preferred method these days is to base the accrued interest on the cumulative movements in the associated inflation index. This calculation method was initiated by Canada for inflation-linked bonds[14] and has been adopted in continental Europe and the US. The UK has announced that it will switch the calculation method for all inflation-linked gilts issued going forward. The method computes (daily) reference numbers for dates using a linear interpolation of the index values of, typically, two and three months ago. The reference number for the first of any calendar month equals the index value of the calendar month three months earlier. $I$(01-May-04)=$CPI$(Feb-04), $I$(01-Jun-04)=$CPI$(Mar-04), and so on. The reference numbers for other dates can then be computed using linear interpolation of the reference numbers of the first days of the calendar months. For example, in Figure 4.4 we compute the reference number, $I$(12-May-04) at 12 May 2004 for the US CPI index. In general, the daily reference number can be computed as follows:

$$I(dd/mm/yy) = I(01/mm/yy) + \frac{dd-1}{DiM}\left[I(01/mm+1/yy) - I(01/mm/yy)\right],$$

where $DiM$ denotes the number of days in the month for all days between the first of January and the first of December. For the days in December we have:

$$I(dd/12/yy) = I(01/12/yy) + \frac{dd-1}{DiM}\left[I(01/01/yy+1) - I(01/12/yy)\right].$$

**Figure 4.4. Daily reference numbers calculation for US CPI**




*In this figure we compute the daily reference number for the US CPI index for 12 May 2004.*
*Source: Lehman Brothers.*

*Bonds are quoted in real terms, but need to be paid in nominal terms*

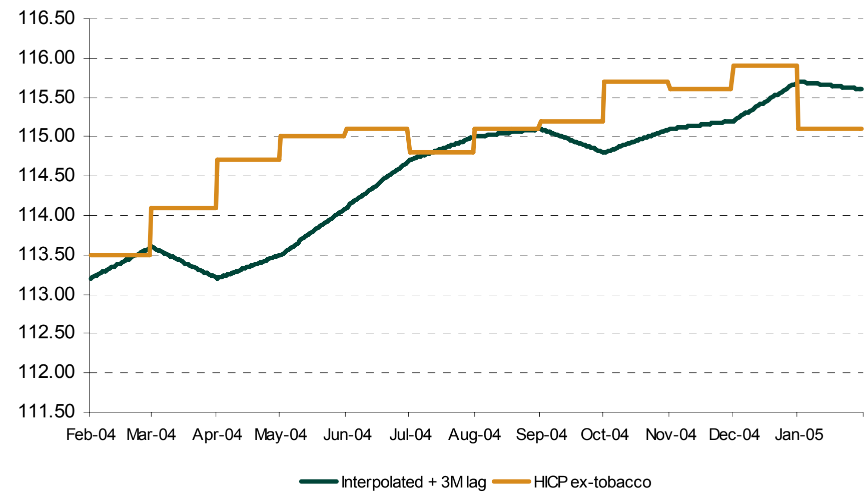
Using the (daily) reference numbers, inflation-linked bonds can be quoted in the standard manner, that is, as real bonds. However, as mentioned above, in order to get the value of the inflation-linked bond, this price in real terms should be multiplied by the index ratio which is the current daily reference number computed in the manner suggested by the Canadian Treasury divided by the daily reference number at the start of the bond.

In Figure 4.5 we plot the index value for the HICPxT market and the linearly interpolated reference numbers associated with the OAT€i. In order to get the relative

[13] *See the document "How to calculate cash flows on inflation-linked gilts" on http://www.dmo.gov.uk/gilts/indexlink/f2lin.htm for detailed examples.*
[14] *Canadian Real Return Bonds (RRBs).*

daily reference numbers for the OAT€is, the reference numbers need to be divided by the base index for the particular OAT€is.[15]

**Figure 4.5. HICPxT and associated daily reference numbers**




*The solid green line represents the interpolated index numbers for OAT€is and the dashed gold line represents the HICPxT level. Note that the solid line lags the dashed line by three months.*
*Source: Agence France Trésor; Lehman Brothers.*

## 4.4. Breakeven inflation

To explain the concept of breakeven inflation, we consider two products available in the market today. The first is a nominal zero-coupon bond with maturity date $T$, whose nominal value today is indicated by $D_n(0,T)$ and pays off $1$ at maturity. The second is a zero-coupon inflation-linked bond with maturity date $T$, whose nominal value today we indicate by $D_{IL}(0,T)=I(0)D_r(0,T)$, where $I(0)$ denotes the current reference number and $D_r(0,T)$ denotes the real value of a real bond with maturity date $T$. The final payoff of this inflation-linked bond at maturity will equal $I(T)$, the reference number at maturity.

We assume an investor has €100 to invest and needs to choose between the following two investments. Investment 1 is in nominal zero-coupon bonds, while investment 2 is in inflation-linked zero-coupon bonds.

1. Invest €100 in zero-coupon nominal bonds, that is, $100/D_n(0,T)$ units. The nominal payoff of this investment at maturity is given by:

$$\frac{100}{D_n(0,T)}\times 1 = 100(1+y_n(0,T))^T,$$

   where $y_n(0,T)$ is the annualised nominal yield on the nominal zero-coupon bond. Assuming $D_n(0,T)=0.9615$ with $T=1$, we have a final payoff of:

$$\frac{100}{0.9615}\times 1 = 104.00.$$

2. Invest €100 in zero-coupon inflation-linked bonds, that is, $100/(\,I(0)D_r(0,T)\,)$ units. The nominal payoff of this investment at maturity is given by:

$$\frac{100}{I(0)D_r(0,T)}\times I(T) = 100(1+i(0,T))^T(1+y_r(0,T))^T,$$

[15] *For the OAT€i 25 July 2012 this is 108.98710; for the OAT€i 25 July 2032 it is 111.15484; and for the OAT€i 25 July 2020 it is 112.62258.*

where i(0,*T*) denotes the annual realised inflation and $y_r(0,T)$ is the annualised real yield on the real bond. Assuming $I(0)=100$ and $D_r(0,T)=0.9804$ for $T=1$ we have as a final payoff:

$$\frac{100}{98.04} \times I(T) = 102.00 \times (1 + i(0,T)),$$

which will depend on the inflation realised in the next year.

The returns are illustrated in Figure 4.6.

**Figure 4.6. Nominal versus inflation-linked investment**

| | **Today** | | **Maturity** |
|---|---|---|---|
| Nominal | 100 | $\longrightarrow$ | $100(1+y_n(0,T))^T$ |
| Inflation-linked | 100 | $\longrightarrow$ | $100(1+y_r(0,T))^T(1+i(0,T))^T$ |

*This figure presents the payoffs of a nominal and inflation-linked zero-coupon bond. The payoff from the nominal investment can be contracted today, while the payoff from the inflation-linked bond depends on realised inflation.*

For the nominal investment, the nominal payoff at maturity is known today as $D_n(0,T)$, and thereby $y_n(0,T)$ are known today. For the inflation-linked investment, the nominal payoff at maturity depends on the realised inflation from today to maturity, $i(0,T)$. If realised inflation, $i(0,T)$, turns out to equal:

$$\frac{1+y_n(0,T)}{1+y_r(0,T)} - 1 = \frac{1.04}{1.02} - 1 = 1.97\%$$

the investor would, ex-post, be indifferent between investment 1 and 2. We define this quantity as the breakeven inflation rate, $b(0,T)$:

$$b(0,T) = \frac{1+y_n(0,T)}{1+y_r(0,T)} - 1 \cdot$$

*Breakeven inflation gives the inflation rate that makes an investor indifferent between nominal and inflation-linked investments*

It is easy to check that if inflation equalled 1.97% investors would have been indifferent between investing in the inflation-linked and the nominal bond. In the case of the nominal bond, they would have invested €100 in 100/0.9615=104.00 nominal bonds, which returned €104.00 at maturity. In the case of the inflation-linked bond, they would have invested €100 in 100/0.9804=102.00 inflation-linked bonds resulting in 102.00×101.97 = €104.00 at maturity. The payoffs in both nominal and real terms thus coincide for both the nominal and inflation-linked bond if realised inflation equals the breakeven inflation. If the realised inflation turns out to be higher (lower) than $b(0,T)$, investors would have been better off investing in the inflation-linked (nominal) bond.

*The breakeven reference number gives the index level that makes an investor indifferent between nominal and inflation-linked investments*

The breakeven rate gives us the indifference point of the realised inflation rate between the inflation-linked and the nominal investment. Another quantity of interest is the reference level for which the investor would be indifferent. This reference level is called the breakeven reference number and is denoted by $I(0,T)$. If the reference level at maturity, $I(T)$, equals:

$$I(0,T) = \frac{I(0)D_r(0,T)}{D_n(0,T)} = I(0)(1+b(0,T))^T = 100 \times (1.0197) = 101.97$$

the investor would, *ex post*, be indifferent between investment 1 and 2. If the reference index at maturity turns out to be higher (lower) than $I(0,T)$, investors would have been better off investing in the inflation-linked (nominal) bond.

With the introduction of the breakeven reference level, we can write the current nominal value of an inflation-linked payment at time $T$ as $D_n(0,T)\times I(0,T)$, the discounted nominal value of the breakeven reference number. This follows from the fact that, by definition, we have:

$$I(0,T)D_n(0,T) = I(0)D_r(0,T).$$

Below we summarise the notation introduced in this section.

**Summary of notation introduced**

| | |
|---|---|
| $b(0,T)$ | Breakeven inflation rate for inflation period from today to $T$. |
| $I(0,T)$ | Breakeven reference number for time $T$ seen from today. |

### 4.5. Components of breakeven inflation rate

It is tempting to say that the breakeven rate should equal expected inflation.[16] However, although expected inflation typically comprises the largest component of the breakeven swap rate, there are several reasons why they will not usually be the same.

1. First, there is the *compounding effect*, which is a mathematical point. If the annualised inflation for the period from 0 to $T$, that is $i(0,T)$, is random, the expected payoff of an inflation-linked security would be higher than if it were to grow at the expected annualised inflation rate. In formulas, the compounding effect can be presented as:

$$E\left[(1+i(0,T))^T\right] \geq (1+E[i(0,T)])^T,$$

where $E$ denotes expectation. The equality only applies if $i(0,T)$ is deterministic.[17] Thus, the compounding effect has upward pressure on breakeven inflation rates. Furthermore, the difference between breakevens and expected inflation will be higher at times of higher volatility. For 5-year inflation swaps the effect is about 3 basis points if the expected inflation equals 2% with a 0.5% standard error and about 6 basis points for a 30-year swap.

2. Second, the inflation convexity, meaning the second-order price effect in case of inflation changes, increases with the maturity of the bond. High convexity is attractive for investors: it means that prices rise more than inflation duration predicts if breakeven inflation rates increase, and decrease less than inflation duration predicts if breakeven inflation rates decrease. As convexity is attractive for investors, it pushes down the breakeven rates.

3. Finally, as inflation-linked bonds provide a high degree of real value certainty, investors are willing to pay an *inflation risk premium* to receive inflation. The inflation risk premium pushes breakeven inflation higher than expected inflation. Let us explain this in more detail.

We consider risk-averse investors who are interested in real income which is perfectly matched by the daily reference numbers, $I$.[18] At time $t$ these investors can invest in either an inflation-linked bond with maturity $T$ offering them a real return of $y_r(0,T)$ or a nominal bond with maturity $T$ offering them a nominal return of $y_n(0,T)$. This gives the following real returns of the nominal and the inflation-linked bond, respectively.

---

[16] *Note for the technical reader: The breakeven swap rate is the expected inflation under the T-forward pricing measure. However, this expectation is generally not equal to the expectation under the physical (real-world) measure.*

[17] *The inequality is a special case of Jensen's inequality which says that the expectation of a convex function is larger than the function evaluated at the expectation.*

[18] *As discussed in Section 4.3 this is not strictly the case, but a reasonable approximation.*

$$\frac{I(0)(1+y_n(0,T))^T}{I(T)} \quad \text{versus} \quad (1+y_r(0,T))^T$$

Because the real return on the nominal bond is uncertain and the real return on the inflation-linked bond is certain, risk-averse investors will only consider investing in the nominal bond if they are compensated for bearing the inflation risk.[19] This will be the case if the expected real return on the nominal bond is higher than the real return on the inflation-linked bond, or if the nominal return on the nominal bond is higher than the expected nominal return on the inflation-linked bond, i.e.

$$(1+y_n(0,T))^T \geq (1+y_r(0,T))^T E\left[\frac{I(T)}{I(0)}\right].$$

The additional return that sovereigns (or other issuers) need to pay on nominal issues compared with inflation-linked issues is called the inflation risk premium, which we denote by $p(0,T)$. We can now write the nominal rate as a Fisher equation (after the American economist Irving Fisher):[20]

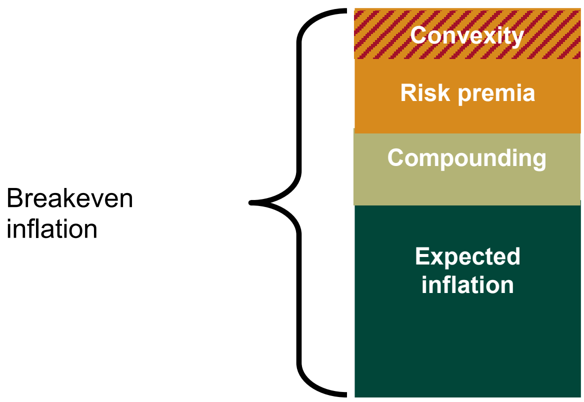
$$1+y_n(0,T) = (1+y_r(0,T)) \times (1+E[i(0,T)]) \times (1+c(0,T)) \times (1+p(0,T)).$$

*Empirical evidence suggests that sovereigns can save substantial amounts by issuing inflation-linked debt*

Thus the nominal return equals the real return times the expected index increase times the risk premium. The size of the inflation risk premium depends on the volatility of inflation (higher volatility leads to higher premium) and the risk-averseness of investors (the more risk-averse the higher the premium). It is hard to put a number on the inflation risk premium and the results differ according to market and study (see, for example, Campbell and Shiller (1996), Gong and Remolona (1996) for US data and Foresi *et al.* (1997) for UK data among others). These studies suggest that sovereigns can save substantial amounts by issuing inflation-linked debt.

Figure 4.7 shows the components of the breakeven inflation rate.

**Figure 4.7. Components of breakeven inflation**




*Note that the scaling should not be taken indicative of the actual size.*
*Source: Lehman Brothers.*

[19] *This is not strictly true as it could be that nominal debt offers diversification benefits that inflation-linked bonds do not have with the rest of the investor's portfolio. This is unlikely (typically even the other way) and therefore we ignore that possibility.*

[20] *Note that $E[1+i(0,T)]^T \geq [1+E\,i(0,T)]^T$ and we have written $E[1+i(0,T)]^T = [1+E\,i(0,T)]^T\,[1+c(0,T)]^T$, where $c(0,T) \geq 0$ denotes the compounding effect. Furthermore, we raised both sides of the equation to the power of 1/T and the risk premia $p(0,T)$ is such that the equality sign applies.*

## 5. ZERO-COUPON INFLATION SWAPS

*Description*

*Inflation swaps can be used to hedge out inflation risk*

The zero-coupon inflation swap has become the standard inflation derivative. For many, it is the basic building block of the inflation derivatives market. Its appeal is its simplicity and the fact that it offers investors and hedgers a wide range of possibilities that did not previously exist in the cash market.

A fixed zero-coupon inflation swap is a bilateral contract that enables an investor or hedger to secure an inflation-protected return with respect to an inflation index. The inflation buyer (also called the inflation receiver) pays a predetermined fixed rate, and in return receives from the inflation seller (also called the inflation payer) inflation-linked payment(s).

*Inflation swaps can be spot or forward starting*

The mechanics are fairly simple; today an inflation payer and an inflation receiver agree to exchange the change in the inflation index value from a base month (e.g. November 2004) to an end month (e.g. November 2009) versus a compounded fixed rate. If the value of the index in the base month is known at the time of the inception of the contract, we call the inflation swap *spot starting*. If the value of the index in the base month is not yet known, we speak of a *forward starting* inflation swap. Figure 5.1 gives an example term sheet of a spot starting inflation swap for the European HICPxT market.

**Figure 5.1. Example of a term sheet for HICPxT zero-coupon inflation swap**

| | |
|---|---|
| Notional: | €100,000,000 |
| Index: | HICPxT (non revised) |
| Source: | First publication by Eurostat as shown on Bloomberg CPTFEMU |
| Trade date: | 10 February 2005 |
| Start date: | 12 February 2005 |
| End date: | 12 February 2010 |
| First fixing: | 115.60 (November 2004) |
| Fixed leg: | $(1+2.11\%)^5$ -1 |
| Inflation leg: | $\frac{HICPxT(Nov/09)}{HICPxT(Nov/04)}-1=\frac{I(01-\text{Feb}-10)}{I(01-\text{Feb}-05)}-1$ |

*The euro market trades monthly index levels*

We see that the inflation swap starts on 12 February 2005 and ends on 12 February 2010 with the exchange of cashflows. As the value of the HICPxT index in the contract month of 2009 needs to be known at the payment, the contract month is lagged to the current month (usually by two to three months). In the above example, the contract month is November, and the HICPxT index values for November are normally published mid-December, which is well before the payment/end date. As the value of the HICPxT index in November 2004 (it equalled 115.60) is known by 12 February 2005, the inflation swap in our example is spot starting. It is market standard to quote fixed inflation swaps whose initial lifetime equals a multiple of whole years (5 years in our example). Note that as the contract month is November, the inflation leg payoff in terms of reference numbers is based on 1 February not 12.February.[21] This has the advantage that all contracts trading with the same contract month and maturity have the same final payoff. This simplifies closing out of the position.

[21] *Recall that for the HICPxT index the reference numbers are three months lagged.*

Not all markets use the convention to pay in terms of index levels. The market standard for the French FR-CPI and US-CPI is to define the payout on the inflation leg in terms of reference numbers. A term sheet would look like the one shown in Figure 5.2.

**Figure 5.2. Example of a term sheet for US CPI zero-coupon inflation swap**

| | |
|---|---|
| Notional: | \$100,000,000 |
| Index: | US CPI-NSA (non-revised) |
| Source: | First publication by BLS as shown on Bloomberg CPURNSA |
| Trade date: | 10 February 2005 |
| Start date: | 12 February 2005 |
| End date: | 12 February 2010 |
| First fixing: | 190.77500 |
| Fixed leg: | $(1+2.75\%)^5 - 1$ |
| Inflation leg: | $\frac{I(12-\mathrm{May}-10)}{I(12-\mathrm{May}-05)} - 1 = \frac{\frac{20}{31} CPI(Feb/10) + \frac{11}{31} CPI(Mar/10)}{\frac{20}{31} CPI(Feb/05) + \frac{11}{31} CPI(Mar/05)} - 1$ |

*The US market trades reference numbers*

Figure 5.3 gives an overview of the market conventions used for trading zero-coupon swaps in the different indices. The convention and lag are also indicated on the LehmanLive page for inflation products.

**Figure 5.3. Market conventions for zero-coupon swaps**

| Market | Method |
|---|---|
| European (HICPxT) | Monthly index level (3M lag) |
| French (FR CPI) | Interpolated values |
| United Kingdom (UK RPI) | Monthly index level (2M lag) |
| United States (CPI-NSA) | Interpolated values |

*Source: Lehman Brothers.*

*Less liquid markets are typically quoted as spreads to the liquid markets*

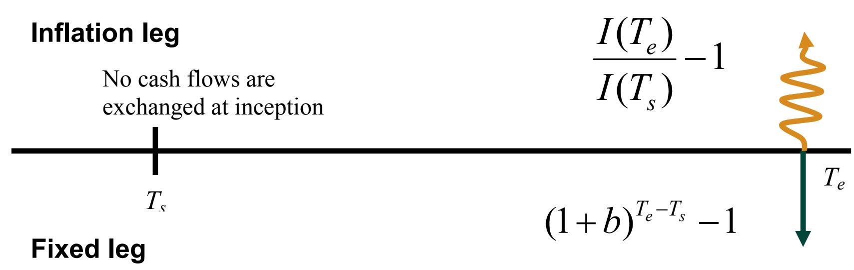
The less liquid markets, such as the Belgian Health index, are typically quoted as a spread to the HICPxT. For example, if the 5y breakeven rate for HICPxT equals 2.11% and the Belgian Health index is quoted at 50bp, this means that the 5y breakeven rate for the Belgian Health index equals 2.61%.

### Valuation of zero-coupon swaps

An inflation swap has an inflation period[22] starting at $T_s$ and ending at $T_e$ over which the inflation is computed and a single payment date, which we assume to equal $T_e$ for now, when the inflation payment (on the inflation leg) is exchanged with a fixed amount (on the fixed leg). The cashflows are presented in Figure 5.4.

[22] *The dates refer to the dates when the reference numbers are observed. For some markets (most notably the European HICPxT market), it is customary to trade on inflation months, but one can always rewrite this in terms of reference dates and numbers (see Section 4.3 for the relation between reference numbers and index levels).*

**Figure 5.4. Cash flows of zero-coupon inflation swap**




*The breakeven rate depends on the inflation period*

The inflation leg thus pays the net increase in reference numbers from $T_s$ to $T_e$, where I($T_s$) is known. The fixed leg pays a fixed amount which is conveniently written as an accumulated rate, $b$. The rate b is quoted in the market and called the breakeven swap rate. The rate b will differ depending on the current time and the inflation period, and therefore we use the notation $b = b(0;T_s,T_e)$ for the breakeven swap rate today for an inflation period $T_s$ to $T_e$. In general, Ts can be different from today. Based on the term sheet in Figure 5.1 we take $T_s$ = 01-Feb-05 and $T_e$ = 01-Feb-10 and assume today is given by the 10 February 2005.[23] The breakeven inflation swap rate quoted in the market equals $b$(0;01-Feb-05, 01-Feb-10) = 2.11% and the discount factor for 12 February 2010 equals 0.86. Assuming a notional equal to €1,000,000 the value of the fixed leg can then be computed as:

$$\begin{aligned}\text{current value of fixed leg} &= D_n(0,T_e)\left[(1+b(0;T_s,T_e))^{T_e-T_s} - 1\right] \\ &= 0.86 \times \left[(1+2.11\%)^5 - 1\right] \times 1{,}000{,}000 = 94{,}640.45\end{aligned}$$

The cashflow at maturity remains constant and therefore the fixed leg only varies with the discount factor.

At inception the breakeven swap rate is set at such a level that the market considers the value of the fixed leg to equal the value of the uncertain inflation leg:

$$\text{current value of inflation leg} = \text{current value of fixed leg} = 94{,}640.45.$$

The only unknown on the inflation leg is the reference number at $T_e$. Using the concept of the inflation-linked zero-coupon bond introduced in Section 4.2 we know that the current value of a payoff of $I(T_e)$ at $T_e$ equals $I(0)D_r(0,T_e)$. This allows us to write the current value of the inflation leg as follows:

$$\text{current value of inflation leg} = \left[\frac{I(0)D_r(0,T_e)}{I(T_s)} - D_n(0,T_e)\right] \times 1{,}000{,}000 = 94{,}640.45$$

where the real discount bond, $D_r(0,T_e)$, is the remaining unknown. Using the fact that the value of the fixed leg and inflation leg are equal at inception, we find that the value of $D_r(0,T_e)$ consistent with the quoted breakeven swap rate equals:

$$\begin{aligned}D_r(0,T_e) &= \frac{I(T_s)}{I(0)} D_n(0,T_e)\left(1+b(0;T_s,T_e)\right)^{T_e-T_s} \\ &= \frac{115.60}{115.70} \times 0.86 \times (1+2.11\%)^5 = 0.954,\end{aligned}$$

where the value of the HICPxT index for November 2004 equals 115.60, the reference number today (12 February 2005) equals 115.70, and as before the discount factor for 12 February 2010 equals 0.86.

Besides a breakeven swap rate for the swap, we can also compute a breakeven reference number for the zero-coupon inflation swap which we denote by $I(0;T_s,T_e)$. It is given by:

[23] *Note that the reference number at 01-Feb-05 equals the HICPxT for November 2004.*

$$I(0;T_s,T_e) = I(T_s)\left(1+b(0;T_s,T_e)\right)^{T_e-T_s}$$
$$= 115.60 \times (1+2.11\%)^5 = 128.32.$$

*The breakeven reference numbers only depend on the maturity date*

It is also easy to show that the start date of the period does not matter. Plugging in the bootstrapped value for $D_r(0,T_e)$ gives:

$$I(0,T_e) = \frac{I(0)D_r(0,T_e)}{D_n(0,T_e)} = I(T_s)\left(1+b_e(0;T_s,T_e)\right)^{T_e-T_s} = I(0;T_s,T_e).$$

One can also check that $I(0)D_r(0,T)=I(0,T)D_n(0,T)$. We have 115.70×0.954 = 128.32×0.86. As a special case of our extended definition of the breakeven swap rate, we have:

$$b(0,T) = b(0;0,T)$$

for a zero-coupon inflation swap with inflation period from today (t=0) to $T$.

In Section 6.1 we explain how one can mark-to-market inflation swaps after we explain how to construct inflation curves using the zero-coupon inflation swaps.

## 6. INFLATION ANALYSIS FRAMEWORK

The commoditisation and transfer of inflation risk is one of the major goals of the inflation derivatives market. In order for financial institutions to benefit from this market, an analysis framework is necessary. In this chapter we describe such a framework. We start by constructing an inflation curve given market quotes for breakeven inflation swap rates taking seasonality into account. Furthermore, we present and explain a number of risks related to inflation-linked cashflows.

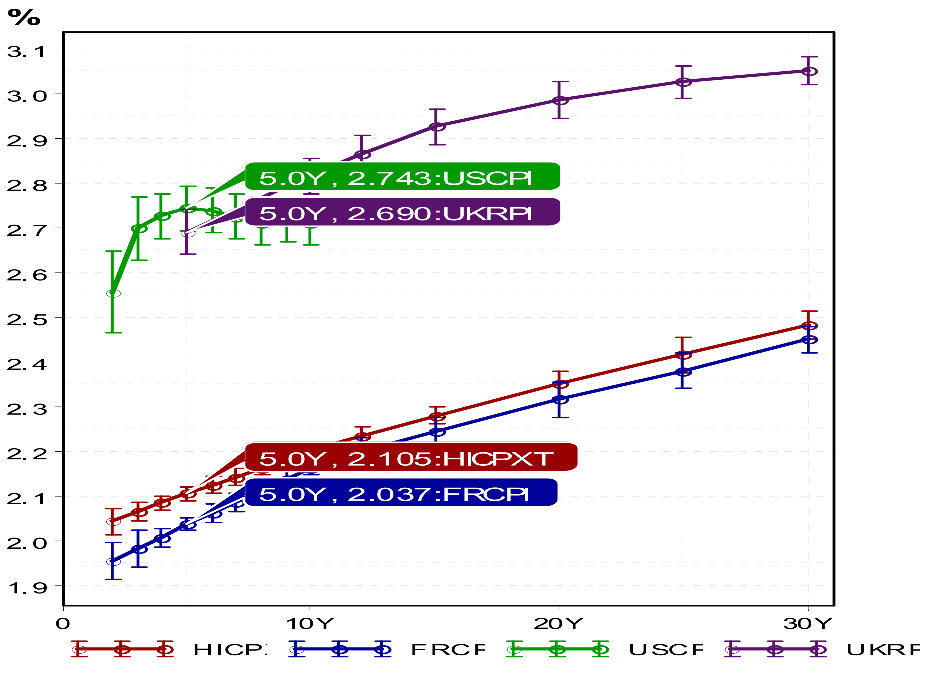
### 6.1. Constructing an inflation curve

In this section we present a methodology to construct inflation-linked swap curves using quotes of breakeven rates for zero-coupon inflation swaps. We also show how one can account for seasonality effects.

*The market quotes inflation swaps with annual tenors*

The market provides bid-offer breakeven inflation rates for several maturities. At the moment broker quotes are typically available for 1,2,..,10,12,15,20,25,30 year maturities. Lehman Brothers provides live quotes for breakeven swap rates on LehmanLive as can be seen in Figure 6.1.

**Figure 6.1. Zero-coupon inflation swap rates on LehmanLive (04-02-2005)**




*Source: LehmanLive.*

The quotes given in Figure 6.1 for the HICPxT index have November as their reference month, whereas for the FRCPI and USCPI quotes are on the reference numbers (Nov-Dec). We focus on the construction of an inflation curve for the HICPxT index. We introduce the following tenor structure $T_0$=01-Feb-05, $T_1$=01-Mar-05,…,$T_{360}$=01-Feb-35. As pointed out in Section 4.3 the reference numbers $I$ are lagged to the inflation index values. In the case of the HICPxT market, the inflation index value for November is equal to the reference number of the first of February. Therefore, for the quotes in terms of reference numbers we have $T_0$ equal to 01-Feb-05. We have seen in Section 4.4 that using the breakeven swap rates in the market we can compute the breakeven reference numbers in the following manner:

$$I(0,T_i) = I(T_0)\left(1 + b(0;T_0,T_i)\right)^{T_i - T_0}.$$

for $i$ = 12, 24, …, 120, 144, 180, 240, 300, 360. Figure 6.2 shows the breakeven inflation index values and breakeven reference numbers for the given breakeven inflation swap rates.

**Figure 6.2. Computing the inflation curve on an annual basis**

| Tenor | Breakeven rate | Maturity inflation index month | Maturity reference date | Breakeven reference number | Unadjusted forwards |
|---|---|---|---|---|---|
| 1Y | 2.04% | Nov-05 | 01-Feb-06 | $115.60\times(1+2.04\%)^{1}$ = 117.96 | 2.02% |
| 2Y | 2.04% | Nov-06 | 01-Feb-07 | $115.60\times(1+2.04\%)^{2}$ = 120.36 | 2.02% |
| 3Y | 2.07% | Nov-07 | 01-Feb-08 | $115.60\times(1+2.07\%)^{3}$ = 122.93 | 2.11% |
| 4Y | 2.09% | Nov-08 | 01-Feb-09 | $115.60\times(1+2.09\%)^{4}$ = 125.57 | 2.13% |
| 5Y | 2.11% | Nov-09 | 01-Feb-10 | $115.60\times(1+2.11\%)^{5}$ = 128.32 | 2.17% |
| 6Y | 2.13% | Nov-10 | 01-Feb-11 | $115.60\times(1+2.13\%)^{6}$ = 131.18 | 2.21% |
| 7Y | 2.14% | Nov-11 | 01-Feb-12 | $115.60\times(1+2.14\%)^{7}$ = 134.07 | 2.18% |
| 8Y | 2.16% | Nov-12 | 01-Feb-13 | $115.60\times(1+2.16\%)^{8}$ = 137.15 | 2.27% |
| 9Y | 2.18% | Nov-13 | 01-Feb-14 | $115.60\times(1+2.18\%)^{9}$ = 140.36 | 2.31% |
| 10Y | 2.20% | Nov-14 | 01-Feb-15 | $115.60\times(1+2.20\%)^{10}$ = 143.70 | 2.35% |
| 12Y | 2.24% | Nov-16 | 01-Feb-17 | $115.60\times(1+2.24\%)^{12}$ = 150.80 | 2.41% |
| 15Y | 2.28% | Nov-19 | 01-Feb-20 | $115.60\times(1+2.28\%)^{15}$ = 162.11 | 2.41% |
| 20Y | 2.35% | Nov-24 | 01-Feb-25 | $115.60\times(1+2.35\%)^{20}$ = 183.96 | 2.53% |
| 25Y | 2.42% | Nov-29 | 01-Feb-30 | $115.60\times(1+2.42\%)^{25}$ = 210.17 | 2.66% |
| 30Y | 2.48% | Nov-34 | 01-Feb-35 | $115.60\times(1+2.48\%)^{30}$ = 241.06 | 2.74% |

*Source: Lehman Brothers.*

The column unadjusted forwards denotes the growth rates in the breakeven reference numbers and are given by log(117.96/115.60), log(120.36/117.96),…, log(241.06/210.17)/5, where log denotes the natural logarithm.

*An interpolation scheme is needed for non-quoted swaps*

For mark-to-market valuation of off-market swaps (for instance, an inflation swap with December as its reference month) we need breakeven reference numbers for all intermediate months as well. As no other market information is available, these have to be determined via an interpolation scheme. We want to construct a breakeven reference curve for 30 years on a monthly basis. A (too) simple approach would be to linearly interpolate the reference numbers given in Figure 6.2. Although this methodology produces a breakeven reference curve consistent with market data, it suffers from two main drawbacks. First, it ignores any seasonal pattern. Second, it produces a rather bumpy breakeven rate curve. In the next section we present an alternative methodology that allows for easy integration of seasonality effects.

### *Incorporating seasonality*

We assume that the breakeven reference numbers are of the following form:

$$I(0,T_i) = I(T_0)\exp\left(\int_{T_0}^{T_i}\left[f(u)+s(u)\right]du\right) \text{ for } i = 1,.., 360.$$

Using a continuously compounded forward inflation rate, $f$, allows us to separate the annual inflation component from the seasonal component in a straightforward manner. We can write:

$$I(0,T_i) = I(T_0)\exp\left(\int_{T_0}^{T_i} f(u)du\right)\times\exp\left(\int_{T_0}^{T_i} s(u)du\right)$$

or in words:

breakeven reference number = start reference number × gross breakeven inflation × seasonal adjustment.

*The breakeven reference numbers can be split into an annual inflation component and a seasonal component*

Several assumptions can be made for the deterministic functions *f(u)*, the instantaneous forward rate, and *s(u)*, the seasonal function. We assume both to be piece-wise constant. We assume the seasonals to be piece-wise constant within a given month, that is:

$$s(u) = \begin{cases} s_1 & \text{for u in January} \\ \vdots & \vdots \\ s_{12} & \text{for u in December} \end{cases}$$

Furthermore, the sum of the seasonal factors should equal zero in order to ensure that there is no seasonal effect for full-year swaps.[24] No knowledge of the inflation curve is needed to compute the seasonal adjustment factors.[25] We show them in Figure 6.3. The seasonal adjustment for November-December is simply the December seasonal, the November-January seasonal is the sum of the December and January seasonal, and so on.

**Figure 6.3. Annualised seasonal adjustment factors**

| Period | Seasonal | Seasonal adjustment | Period | Seasonal | Seasonal adjustment |
|---|---|---|---|---|---|
| November-December | 1.39% | 1.39% | November-June | -0.80% | 4.61% |
| November-January | -2.37% | -0.98% | November-July | -1.86% | 2.75% |
| November-February | 2.22% | 1.24% | November-August | -0.95% | 1.80% |
| November-March | 2.32% | 3.56% | November-September | 0.30% | 2.10% |
| November-April | 1.30% | 4.86% | November-October | -0.65% | 1.45% |
| November-May | 0.55% | 5.41% | November-November | -1.45% | 0.00% |

*Source: Lehman Brothers.*

Given the breakeven reference numbers shown in Figure 6.3, we can compute unadjusted forward inflation rates in Figure 6.4.[26] We have assumed these forward rates to be piece-wise constant.[27] For instance, the unadjusted forward rate for the second year is given by log(120.36 / 117.96) = 2.02%. We can now compute a monthly breakeven reference curve using the following iterative formula:

$$I(0,T_i) = I(0,T_{i-1}) \times \exp\big((T_i - T_{i-1})(f_i + s_i)\big) \text{ for } i = 13,\ldots,360,$$

where $f_i$ denotes the annual inflation (the unadjusted forwards) and $s_i$ the annualised seasonal component for the period [$T_{i-1}$, $T_i$]. For example, we find the breakeven reference number for 1 March 2006 in the following manner:

$$\begin{aligned} I(0,01-\text{Mar}-06) &= I(0,01-\text{Feb}-06) \times \exp\big((2.02\% + 1.39\%)/12\big) \\ &= 117.96 \times \exp\big((2.02\% + 1.39\%)/12\big) = 118.30 \end{aligned}$$

and for 1 April 2006 we find:

$$\begin{aligned} I(0,01-\text{Apr}-06) &= I(0,01-\text{Feb}-06) \times \exp\big((2 \times 2.02\% - 0.98\%)/12\big) \\ &= I(0,01-\text{Mar}-06) \times \exp\big((2.02\% - 2.37\%)/12\big) \\ &= 118.29 \times \exp\big((2.02\% - 2.37\%)/12\big) = 118.26. \end{aligned}$$

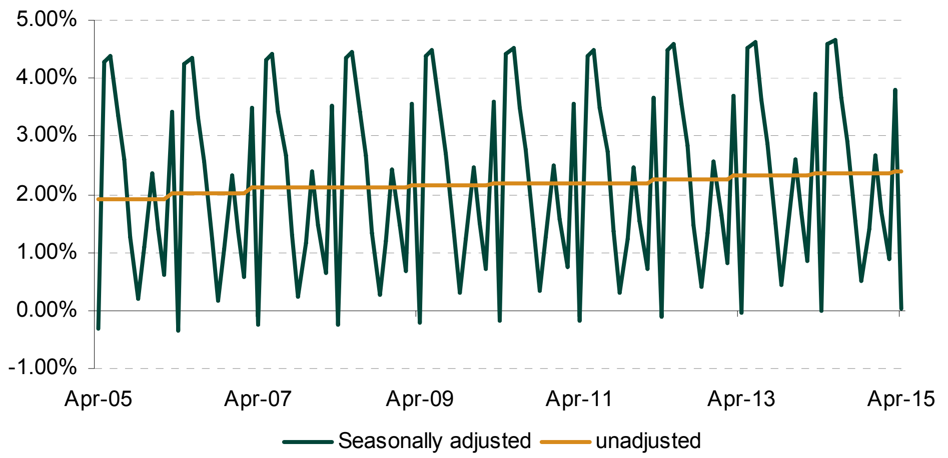
*The first month is typically already known*

The first year is exceptional in that, typically, the breakeven reference number of the next month is already known. In our example, we know that the HICPxT equals 115.90 for December 2004. Therefore, we take as the forward rate for the first year [log(117.96 / 115.90)+1.39%/12]×12/11 = 2.05%. We subtracted the December seasonal as we only have the period January-November remaining and multiply by 12/11 to annualise the

[24] *Since $s_1+\ldots+s_{12}=0$, we have that exp(f)=exp(f+$s_1$+…+$s_{12}$), and so on.*
[25] *If the market matures and swaps traded on several reference months are traded, the seasonal factors can be bootstrapped out of market quotes. Currently, quotes on more than one reference month are rare.*
[26] *In Section 7.1 and Appendix A.4 we discuss why these forward rates need to be corrected to represent the rates for forward starting inflation swaps.*
[27] *The piece-wise flat assumption ignores potential dependence structures between nominal rates, real rates and inflation.*

rate. The seasonal factors we use to adjust the rates are December-January (-2.37%),…, December-November (-1.39%).

**Figure 6.4. Annual forward rates and seasonally adjusted forward rates**




*Source: Lehman Brothers.*

This procedure can be done recursively to construct the whole curve. Figure 6.4 presents the seasonally adjusted forward rates and unadjusted rates for April 2005 to April 2015. In Figure 6.5 we present the construction of the first year of the inflation curve.

**Figure 6.5. Computing the monthly breakeven reference number curve**

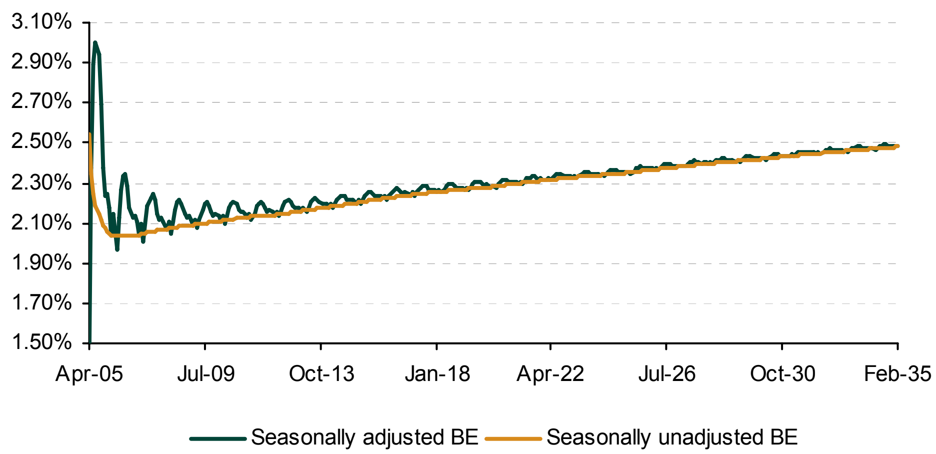
| Breakeven index month | Maturity reference date | y-o-y seasonal | Breakeven Index t-1 | | | Breakeven Index |
|---|---|---|---|---|---|---|
| Nov-04 | 01-Feb-05 | | | | | 115.60 |
| Dec-04 | 01-Mar-05 | 1.39% | 115.60 | | | 115.90 |
| Jan-05 | 01-Apr-05 | -2.37% | 115.90 | × exp([2.05%-2.37%]/12) | = | 115.87 |
| Feb-05 | 01-May-05 | 2.22% | 115.87 | × exp([2.05%+2.22%]/12) | = | 116.28 |
| Mar-05 | 01-Jun-05 | 2.32% | 116.28 | × exp([2.05%+2.32%]/12) | = | 116.71 |
| Apr-05 | 01-Jul-05 | 1.30% | 116.71 | × exp([2.05%+1.30%]/12) | = | 117.03 |
| May-05 | 01-Aug-05 | 0.55% | 117.03 | × exp([2.05%+0.55%]/12) | = | 117.28 |
| Jun-05 | 01-Sep-05 | -0.80% | 117.28 | × exp([2.05%-0.80%]/12) | = | 117.41 |
| Jul-05 | 01-Oct-05 | -1.86% | 117.41 | × exp([2.05%-1.86%]/12) | = | 117.43 |
| Aug-05 | 01-Nov-05 | -0.95% | 117.43 | × exp([2.05%-0.95%]/12) | = | 117.53 |
| Sep-05 | 01-Dec-05 | 0.30% | 117.53 | × exp([2.05%+0.30%]/12) | = | 117.76 |
| Oct-05 | 01-Jan-06 | -0.65% | 117.76 | × exp([2.05%-0.65%]/12) | = | 117.90 |
| Nov-05 | 01-Feb-06 | -1.45% | 117.90 | × exp([2.05%-1.45%]/12) | = | 117.96 |

*Source: Lehman Brothers.*

*Seasonal effects become less important for longer maturities*

In Figure 6.6 the monthly breakeven rates (both seasonally adjusted and unadjusted) are shown. A clear seasonal pattern can be recognised, which dies out over time as the seasonal influence becomes less important.

**Figure 6.6. Breakeven inflation rates**




*Source: Lehman Brothers.*

We are not restricted to using piece-wise flat forward rates. For example, one can apply linear or quadratic forward rates instead of the piece-wise flat forwards. Another possibility is to use (cubic) splines in order to get a smooth curve.

The US and French markets trade zero-coupon swaps based on reference numbers, so we typically have inflation periods which are partially covering months. Although this slightly complicates the calculation of the seasonal-adjustment factor, the curve can be constructed using the same methodology.

*Marking to market inflation swaps*

*Changes in inflation expectations are reflected in the mark-to-market value*

Even though the payout of an inflation swap is linked to realised inflation, it is also an inflation expectations product. On a mark-to-market basis, the value of the inflation swap changes in line with changes in expected inflation as reflected in the changing breakeven inflation rate. This results from the fact that the mark-to-market value of the inflation swap has to reflect the cost of entering into the offsetting transaction. For an inflation buyer, this requires selling inflation on an inflation swap with identical characteristics. The mark-to-market today ($t=0$) for an inflation swap initiated at time $t_B \le 0$ with inflation period from $T_s$ to $T_e$ for the inflation buyer is given by:

$$D_n(0,T_e)\left([1+b(0;T_s,T_e)]^{T_e-T_s} - [1+b(t_B;T_s,T_e)]^{T_e-T_s}\right),$$

where $b(0;T_s,T_e)$ and $b(t_B;T_s,T_e)$ denote the breakeven inflation swap rates at the current time $0$ and at the inception time $t_B$, respectively. The term on the left denotes the value of the inflation leg at time $0$, while the term on the right denotes the original value of the inflation (and fixed) leg. However, it is unlikely that the market is still quoting the original swap structure. Suppose we entered a 5-year inflation swap (say, February 2004 to February 2009) this structure is likely only quoted during May and June 2004. Given an inflation curve consisting of the breakeven reference numbers $I(0,\mathrm{T})$ for $0 \le T \le T_e$, the mark-to-market value of this structure is given by:

$$D_n(0,T_e)\left(\frac{I(0,T_e)}{I(T_s)} - \frac{I(t_B,T_e)}{I(T_s)}\right),$$

where $I(0,T_s)$ and $I(t_B,T_s)$ denote the breakeven reference numbers for $T_e$ (=01-May-09 for the February 2004 to February 2009 inflation swap) today and at the inception time $t_B$. Above we have detailed the construction of an inflation curve consisting of the breakeven reference numbers $I(0,T)$ as a function of $T$.

In general, the breakeven reference number at maturity will depend on the seasonality assumption used in the inflation curve construction. However, in May and June 2005, 4-

year inflation swaps are quoted for the period (February 2005 to February 2009) and the index increase from February 2004 to February 2005 is known. The inflation swap can then be easily marked-to-market by:

$$D_n(0,T_e)\left(\frac{I(T_s^*)}{I(T_s)}\left[1+b(0;T_s^*,T_e)\right]^4-\left[1+b(t_B;T_s,T_e)\right]^5\right),$$

where $T_s^*$ equals 1 May 05. Thus we can get a seasonally independent mark-to-market value on an annual basis.

*Example 6.1*

We consider a zero-coupon inflation swap on HICPxT for February 2004 to February 2009 paying 1 May 2009. Originally, a breakeven rate of 2.10% was paid for the contract on a notional of €100,000,000. We know that the HICPxT for February 2004 equalled 113.50. Using the breakeven reference curve, we find that the breakeven reference for February 2009 equals:

$$125.57\times\exp(\tfrac{3}{12}\times 2.17\%+\tfrac{1}{12}1.24\%)=126.38$$

where 125.57 is the breakeven reference index level for November 2008, 2.17% the piece-wise flat forward rate from November 2008 to November 2009 (see Figure 6.2) and 1.24% the November-February seasonal (see Figure 6.3). The value of the inflation leg at maturity is therefore equal to:

$$100{,}000{,}000\times\left(\frac{126.38}{113.50}-1\right)=11{,}351{,}405.$$

The contracted payment at maturity was given by:

$$100{,}000{,}000\times\left((1+2.10\%)^5-1\right)=10{,}950{,}359.$$

This results in a mark-to-market value of:

$$0.90\times\left[11{,}351{,}405-10{,}950{,}359\right]=360{,}941$$

where 0.90 is the discount factor from today to 1 May 2009 from the perspective of the inflation receiver.

## 6.2. Inflation and interest rate risk

Similar to nominal bonds and swaps, inflation-linked bonds and swaps are sensitive to interest rates. However, contrary to nominal bonds and swaps, inflation-linked bonds and swaps also have explicit inflation risk. We have seen before that an inflation-linked cashflow at time $T_e$ can be written in two manners:

$$I(0)D_r(0,T_e)=I(0,T_e)D_n(0,T_e)$$

or

$$\frac{1}{(1+y_r(0,T_e))^{T_e}}=\frac{(1+b(0,T_e))^{T_e}}{(1+y_n(0,T_e))^{T_e}}$$

Following the formulation on the left-hand side, we can investigate the risk due to changes in the real rate, $y_r(0,T_e)$. Following the formulation on the right-hand side, we can investigate the risk of changes in the nominal rate, $y_n(0,T_e)$, and the risk of changes in the breakeven rate, $b(0,T_e)$. Of course, both approaches should be equivalent and the approach taken is merely a matter of taste. Below, we discuss the risk in terms of inflation and (nominal) interest rate risk. We prefer this approach because the swap market uses the nominal curve and inflation curve as these are easily constructed from market instruments.[28] In this formulation, the payer of inflation on a zero-coupon swap faces the following risks:

1. Inflation risk, that is, changes in the inflation curve.
2. Interest rate risk, that is, changes in the nominal yield curve.

*Inflation PV01*

*The inflation PV01 gives the sensitivity to changes in the inflation curve*

The inflation PV01 measures the sensitivity to changes in the (annually compounded) inflation curve, *ceteris paribus*.[29] It is approximately equal to the change in value due to a 1bp move in the inflation curve. For an inflation swap with inflation period $T_s$ to $T_e$ and maturing at time $T_e$ the inflation PV01 today is given by:

$$\text{InfPV01} = T_e \times \frac{I(0)}{I(T_s)} \times \frac{D_r(0,T_e)}{1+b(0,T_e)}.$$

One can get the inflation duration by dividing the inflation PV01 by the price. In Appendix A.3 we give the inflation PV01 for inflation-linked coupon bonds.

*Nominal PV01*

*The nominal PV01 gives the sensitivity to changes in the nominal curve*

The nominal PV01 measures the sensitivity to changes in the (annually compounded) nominal curve, *ceteris paribus*. For an inflation swap with inflation period $T_s$ to $T_e$ and maturing at time $T_e$ the nominal PV01 today is given by:

$$\text{PV01} = \frac{T_e}{1+y_n(0,T_e)}\left(\frac{I(0)}{I(T_s)}D_r(0,T_e) - \frac{I(t_B,T_e)}{I(T_s)} \times D_n(0,T_e)\right).$$

At inception ($t_B$=0) the term between brackets equals zero (since $I(t_B)D_r(t_B,T_e)=I(t_B,T_e)D_n(t_B,T_e)$), implying that the inflation swap has no interest rate risk at inception. The situation changes as the swap moves in or out of the money. If the inflation breakevens rise, *ceteris paribus*, the swap moves into the money for the inflation receiver (the term between the brackets is positive) since $I(0)D_r(0,T_e)$ has increased, and the inflation receiver has a negative PV01 as the future cashflow becomes less valuable with higher rates.

*Moves in the inflation curve and nominal curve are correlated*

It can be deceptive to look at *ceteris paribus* moves in the inflation and nominal curves because moves in these curves are typically correlated. As inflation and nominal rates tend to be positively correlated, the risk for the buyer of an upward shift in the nominal curve is thus (much) less than based on the PV01. For instance, in the case of a 50% correlation, the risk is half that of a *ceteris paribus* up move in the nominal curve. The correlation can be made a function of time with typically higher correlation for longer maturities. This enables us to capture the empirical fact that long real interest rates are less volatile than short real rates.

The formulas for coupon inflation-linked bonds and their derivation are detailed in Appendix A.3. It also presents formulas for convexity and cross convexity for inflation and interest rate risk. In Figure 6.7 we give an example of a 5-year inflation-linked coupon bond and calculate the inflation and interest rate PV01s.

[28] *Of course, given any two out of the three curves (nominal, inflation, and real) the third one can be constructed using no-arbitrage arguments but the inflation and nominal curves are typically used.*

[29] *Of course, the PV01 can also be defined as the sensitivity of the inflation curve for other compounding frequencies, e.g., continuously compounded or semi-annually. This has a small impact on the formula.*

**Figure 6.7. Inflation PV01 and nominal PV01 example**

| Year | Coupon | Breakeven rate | Breakeven reference number | Cashflows | Nominal yield |
|---|---|---|---|---|---|
| 1 | 2.50% | 2.00% | 1.020 | 2.55 | 4.00% |
| 2 | 2.50% | 2.08% | 1.042 | 2.61 | 4.10% |
| 3 | 2.50% | 2.14% | 1.066 | 2.66 | 4.18% |
| 4 | 2.50% | 2.18% | 1.090 | 2.73 | 4.24% |
| 5 | 2.50% | 2.20% | 1.115 | 114.28 | 4.26% |
| Price | 102.29 | | | | |
| Inflation PV01 | 4.769 | Nominal PV01 | | -4.675 | |
| 1bp bump in *b*'s | 0.04770 | 1bp bump in $y_n$'s | | -0.04674 | |

*Source: Lehman Brothers.*

In the above example we see that if the inflation curve moves up by 1 basis point, the value of the inflation-linked bond prices moves up by 0.04770, while it moves down by 0.04674 if the interest rate curve moves up by 1 basis point.[30]

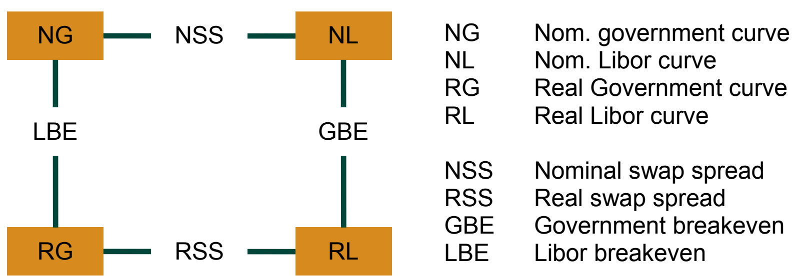
## 6.3. Counterparty risk

So far, we have ignored the credit quality when discussing inflation-linked bonds and inflation swaps to avoid notational complexity. In this section, we take the credit quality explicitly into account and explain the different credit quality curves.

*Inflation swap cashflows have a different credit quality than inflation-linked bond cashflows*

Typically inflation derivatives dealers receive inflation from inflation-linked bonds (the standardised products) and pay inflation on structured inflation derivatives (typically swaps). This introduces a certain type of counterparty risk. Inflation cashflows from governments should be discounted from the inflation-linked government curve, while inflation cashflows from swap counterparties should be discounted from the Libor curve (plus/minus a spread depending on the counterparty's credit quality).[31] Typically, this means that inflation swaps should be discounted off the Libor curve and inflation-linked bonds should be discounted off the government curve. Let us for now assume that all curves are liquid along the whole curve. This is a reasonable assumption on the nominal side, but hardly on the inflation-linked curves as we discuss later. Figure 6.8 gives an example of the relations between the curves.

**Figure 6.8. Government versus swap curves**




*Schematic overview of the relations between the nominal government, nominal Libor, real government, and real Libor curve.*
*Source: Lehman Brothers.*

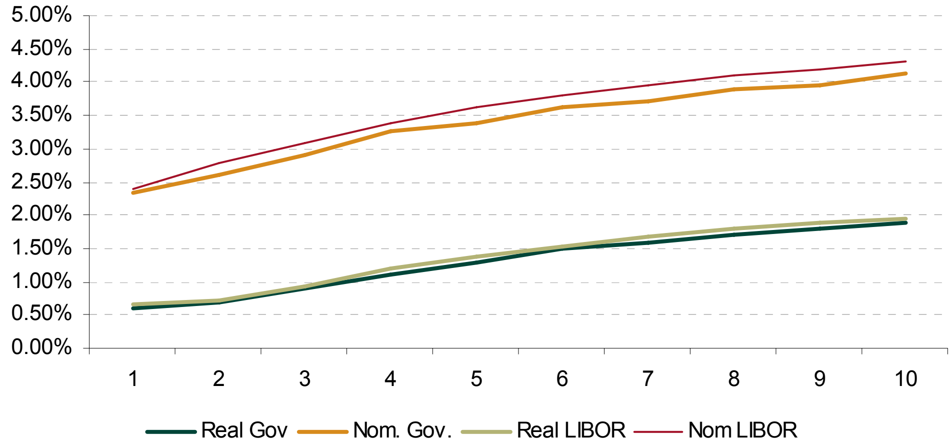
[30] *If we had computed inflation and interest rate PV01s on continuously compounded rates they would be exactly offsetting.*
[31] *Libor is typically taken as having AA-credit quality.*

We denote the four curves above as[32]

$$D_r^G(t,T), y_r^G(t,T) : \text{real government bond curve}$$
$$D_n^G(t,T), y_r^G(t,T) : \text{nominal government bond curve}$$
$$D_r^L(t,T), y_r^G(t,T) : \text{real LIBOR curve}$$
$$D_n^L(t,T), y_r^G(t,T) : \text{nominal LIBOR curve}$$

where $D$ stands for the zero-coupon bond and $y$ for the yield. Figure 6.9 gives an example of the four curves.

**Figure 6.9. Government and Libor example curves**




*Source: Lehman Brothers.*

Using our definition of breakeven inflation, we can now compute the breakeven inflation for both the government curve and the Libor curve. The breakeven reference number and breakeven rate for the Libor inflation curve are given by:

$$I^L(0,T) = \frac{I(0)D_r^L(0,T)}{D_n^L(0,T)} \text{ and } 1+b^L(0,T) = \frac{1+y_n^L(0,T)}{1+y_r^L(0,T)}$$

and for the government inflation curve:

$$I^G(0,T) = \frac{I(0)D_r^G(0,T)}{D_n^G(0,T)} \text{and } 1+b^G(0,T) = \frac{1+y_n^G(0,T)}{1+y_r^G(0,T)}.$$

If we assume that the nominal swap spread equals the real swap spread,[33] that is:

$$\frac{1+y_n^G(0,T)}{1+y_r^G(0,T)} = \frac{1+y_n^L(0,T)}{1+y_r^L(0,T)},$$

we get a unique breakeven reference number, $I^G(0,T) = I^L(0,T)$.

As mentioned earlier, the assumption that both a real government and real swap curve exist is quite strong in the current market environment. In practice, it is unlikely that the nominal and real swap spreads are the same. From an historical perspective it was interesting to see what happened when Italy issued the BTP€i 2008 in 2003. At that time 5y inflation was mainly driven by demand from inflation receivers. This drove up inflation breakeven rates in the swap market, allowing Italy to issue at an attractive level.

[32] *Note that this distinction in notation for credit quality is only made in this paragraph. In other paragraphs we only indicate it if it leads to ambiguity.*

[33] *The spread is based on annual compounding and is only approximately equal to* $y_n^X(0,T) - y_r^X(0,T)$ *for X=G,L. For continuous compounding this would be exact.*

### 6.4. Rounding risk

A typical risk for inflation-linked securities is the rounding risk. Whereas interest rates can take any value, indices are set up to one decimal. Consider the following example.

*Example 6.2*

The value of the HICPxT index is 115.60 for November 2004 and the breakeven rate for a 2-year zero-coupon inflation swap is given by 2.11%. This implies a breakeven HICPxT index level for February 2006 equal to:

$$115.60 \times (1 + 2.11\%)^5 = 128.321.$$

*Inflation indices are rounded to one decimal*

Due to the one decimal place rounding, the index cannot set at this level. Even if the breakeven index realises, the publisher (in this case Eurostat) will round the value up or down to the nearest decimal. In this example, that would be 128.30 or 128.40. The rounding risk decreases quickly with increasing maturity and becomes negligible for long maturities.

Rounding risk is especially important for inflation caps and floors. For instance, if we have a cap with a strike of 2.11%, the rounding can determine whether the cap will end in or out of the money.

### 6.5. Seasonality risk

We saw in Section 3.6 that inflation series have strong seasonal patterns. In this section, we analyse the consequences of these seasonal patterns for pricing and risk management.

*Seasonal effects are not known*

In Section 6.1 we constructed an inflation curve that incorporates seasonality patterns. For the construction we assumed a set of deterministic seasonal factors. In reality, the seasonal pattern is probably not deterministic, or at least not known with certainty as the standard errors in Figure 3.8 indicate. Therefore, inflation products are sensitive to seasonality risk.

*The market provides little information on seasonal effects*

Currently, little market information is available on seasonals. Quotes are rarely given for off-market swaps, although this might change with the introduction of the futures market. There is likely to be a discrepancy between the expected seasonals and the market implied seasonals due to market positioning.[34] For instance, April and May Euro inflation swaps can be hedged by banks using OAT€is, whereas, say,. March inflation swaps cannot (yet) be sourced from the inflation cash market.

*Example 6.3*

We start by discussing the seasonal risk using Example 6.1 based on the data in Figures 6.2 and 6.3. The trade of interest is again a zero-coupon inflation swap on HICPxT for February 2004 to February 2009 payable on 1 May 2009. In order to value the February inflation swap, we need the breakeven reference number for February 2009. We compute the breakeven reference number as:

$$I(0, 01-\text{May}-09) = 125.57 \times \exp\left(\tfrac{3}{12} \times 2.17\% + \tfrac{1}{12}(s_{12} + s_1 + s_2)\right) = 126.383845,$$

where 125.57 is the breakeven reference index level for November 2008, 2.17% the piece-wise flat forward rate from November 2008 to November 2009 (see Figure 6.2) and $s_{12} + s_1 + s_2 = 1.24\%$ is given by our seasonal assumption in Figure 6.3.

This gives rise to the question of the sensitivity of the valuation for the seasonal function. We have seen before that seasonals need to add up to zero on an annual basis. Therefore, if a certain seasonal component moves up at least one of the others needs to move down. We denote the changes in the seasonal function by $\delta_1,\ldots,\delta_{12}$, which need to sum to zero in order to have the changed seasonals to add up to zero. Thus, the adjusted seasonal components are given by:

[34] *Note for the technical reader. The seasonals under the physical measure differ from the seasonals under the risk-neutral measure.*

$$s_1^* = s_1 + \delta_1, ..., s_{12}^* = s_{12} + \delta_{12}.$$

One way to investigate the seasonality risk is by bumping one seasonal up by $x$ basis points and decreasing the remaining seasonals by an equal amount ($x$/11bp) such that the annual zero constraint is not violated.

In our example, we bump the seasonal of each month by 50bp (approximately 1 std. deviation) and decrease the remaining seasonals by 50/11bp and investigate the impact on the P&L of the contract. For instance, if we bump the January seasonal with 50bp and decrease all other months by 50/11bp we get:

$$s_1^* = s_1 + 0.005, ..., s_{12}^* = s_{12} + 0.005/11$$

This gives $s^*_{12} + s^*_1 + s^*_2 = 1.65\%$ and our adjusted breakeven reference number is given by

$$I'(0,01-\text{May}-09) = 125.57 \times \exp(\tfrac{3}{12} \times 2.17\% + \tfrac{1}{12} 1.65\%) = 126.426937.$$

The P&L impact of such a change in the seasonal assumption is given by:

$$100{,}000{,}000 \times \left[\left(\frac{126.43}{113.50} - 1\right) - \left(\frac{126.38}{113.50} - 1\right)\right] = 11{,}389{,}372 - 11{,}351{,}405 = 37{,}967.$$

If we would bump the March seasonal by 50bp and decrease the other months by 50/11bp we get $s^*_{12} + s^*_1 + s^*_2 = 1.10\%$ and our adjusted breakeven reference number is given by:

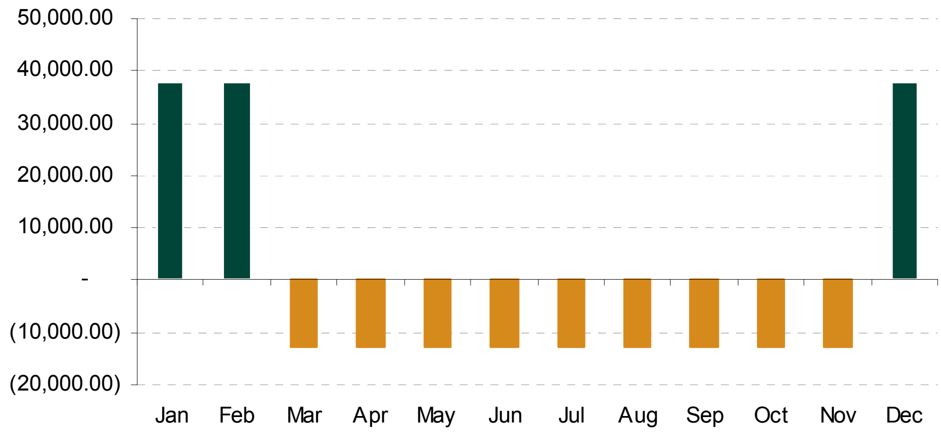
$$I'(0,01-\text{May}-09) = 125.57 \times \exp(\tfrac{3}{12} \times 2.17\% + \tfrac{1}{12} 1.10\%) = 126.369483.$$

The P&L impact of such a change in the seasonal assumption is given by:

$$100{,}000{,}000 \times \left[\left(\frac{126.37}{113.50} - 1\right) - \left(\frac{126.38}{113.50} - 1\right)\right] = 11{,}338{,}752 - 11{,}351{,}405 = -12{,}653.$$

We see in Figure 6.10 that bumping the December to February seasonals has the same impact, which makes sense as only the aggregate of the remaining seasonals impacts the value. The same holds for the seasonals from March to November. As an increase in one of these months decreases the aggregate seasonals from December to February this reduces the value of the swap.

**Figure 6.10. Seasonality risk**




*This figure gives the impact on the P&L of a 1 standard deviation move in the seasonal effect for all months.*
*Source: Lehman Brothers.*

### 6.6. Institutional risk

The inflation indices on which inflation swaps (and bonds) are based are typically defined and published by a statistical bureau. This entails several risks that are difficult to quantify. For example, when additional countries are added to the eurozone, these countries will be incorporated in the European indices. As these countries typically have higher inflation than the current countries, this pushes breakeven rates higher for longer maturities. Another interesting institutional risk is the change of benchmark inflation index for policymakers. For instance, the UK changed its inflation target index from RPIX to UK HICP (denoted CPI by the National Statistics).

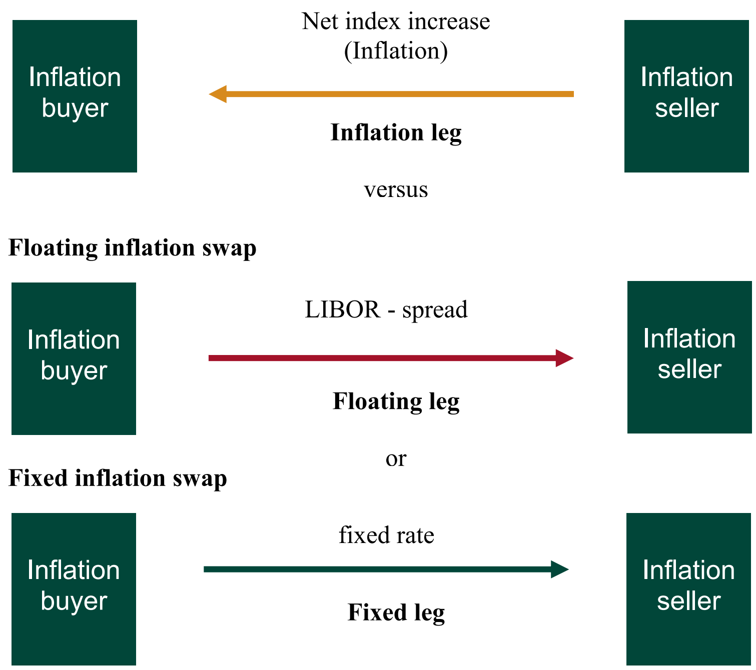
## 7. INFLATION SWAPS AND FUTURES

In Section 5 we introduced and described the (fixed) zero-coupon inflation swap. Besides zero-coupon inflation swaps a number of other inflation swaps are traded, which are typically portfolios of zero-coupon swaps in one way or the other. We describe a revenue inflation swap, an OTC inflation bond, and period-on-period inflation swaps.

*Fixed and floating inflation swaps*

Inflation swaps are bilateral contracts that enable an investor or hedger to secure an inflation-protected return with respect to an inflation index. The inflation buyer (also called the inflation receiver) pays a predetermined fixed or floating rate (usually minus a spread). In return, the inflation buyer receives from the inflation seller (also called the inflation payer) inflation-linked payment(s). Two main types of (zero-coupon) inflation swaps exist: *fixed inflation swaps* (inflation versus fixed rate) and *floating inflation swaps* (inflation versus floating rate, usually Libor). The mechanics are shown in Figure 7.1.

**Figure 7.1. Mechanics of fixed and floating inflation swaps**




*Source: Lehman Brothers.*

*An investor pays inflation on a payer inflation swap and receives inflation on a receiver inflation swap*

We call an inflation swap a *payer inflation swap* if you pay inflation and a *receiver inflation swap* if you receive inflation. Using an interest rate swap (IRS), we can find a no-arbitrage relationship between fixed and floating inflation swaps which we call the *inflation swap parity*:

*Floating payer inflation swap = Fixed payer inflation swap + payer fixed-for-floating swap*

*Floating receiver inflation swap= Fixed receiver inflation swap + receiver fixed-for-floating*

### *Applications*

Many applications exist for inflation swaps, which we now summarise.

**Hedging / Asset Liability Management**

- Inflation swaps can be used to hedge concentrations of inflation risk. This is especially useful for pension funds that have inflation-linked liabilities.

- Inflation swaps can be used to hedge inflation exposures that are not traded in the cash market.
- It is impractical for banks to hedge inflation risks using inflation-linked bonds. Even if two-way liquidity can be found, it imposes heavy balance sheet charges. As inflation swaps are off-balance sheet transactions, they avoid the balance sheet charges.

**Investing**

- Inflation swaps are an unfunded way to take a view on inflation. This allows for leverage and helps those with a high funding cost.
- As inflation swaps are customisable over-the-counter products, investors can tailor the inflation exposure to match their precise requirements in terms of index (e.g. many schemes are linked to domestic wage indices), timing, maturity, size, and so on.
- Inflation swaps can be used to go either long or short inflation on a reference index.
- Investors may not be allowed to sell an inflation-linked asset due to regulatory restrictions, but may be allowed to enter in an inflation swap paying inflation.

**Arbitrage / Trading**

- It is usually easier to pay inflation on an inflation swap than to short an inflation-linked asset.
- Traders can take advantage of price dislocation between the cash and derivatives market by buying the cash asset and paying inflation or selling the cash asset and receiving inflation and trading the swap spread to hedge to government vs. Libor curve spread.

## 7.1. Revenue inflation swap

Unlike a zero-coupon inflation swap, a revenue inflation swap has multiple inflation-linked cashflows. At the end of each period (usually a year) it pays the cumulative inflation from a base month up to the period. For instance, an annual revenue inflation swap on HICPxT with start date and end date equal to 12 May 2004 and 12 May 2009 has the cashflows shown in Figure 7.2.

**Figure 7.2. Cashflows of a typical revenue inflation swap**

| **Notional** | **€ 100m** | **Fixed rate:** | **2.69%** |
|---|---|---|---|
| Start date | 12 May 2004 | Rolls | 12th |
| End date | 12 May 2009 | HICPxT February 2004 | 113.50 |
| Fixed leg frequency | Annual, unadjusted | Fixed leg basis | ACT |
| Inflation leg frequency | Annual, unadjusted | Inflation leg basis | BOND (30/360) |

| | **Fixed rate** | **Fixed leg** | **Discount factor** | **Breakeven Index**[35] | **Breakeven Inflation leg** |
|---|---|---|---|---|---|
| 12 May 2005 | $(1+2.69\%)^1-1$ | 2,692,509 | 0.977 | 116.25 | 2,420,000 |
| 12 May 2006 | $(1+2.69\%)^2-1$ | 5,457,513 | 0.948 | 119.34 | 5,144,516 |
| 14 May 2007 | $(1+2.69\%)^3-1$ | 8,296,966 | 0.912 | 122.69 | 8,099,326 |
| 12 May 2008 | $(1+2.69\%)^4-1$ | 11,212,871 | 0.874 | 126.36 | 11,332,008 |
| 12 May 2009 | $(1+2.69\%)^5-1$ | 14,207,287 | 0.834 | 130.50 | 14,973,878 |

*Source: Lehman Brothers.*

[35] *The breakeven index values for February 2005,…, February 2009 or alternatively the breakeven reference numbers for 01 May 2005,…, 01 May 2009.*

The inflation cashflows can be easily constructed using the zero-coupon inflation swaps as building blocks. A revenue inflation swap is, therefore, nothing more than a portfolio of zero-coupon swaps where the fixed payments are transformed to a constant fixed rate paid at the cashflow dates.

## 7.2. OTC inflation bond

*An OTC inflation bond has cashflows similar to inflation-linked bonds and a credit quality similar to inflation swaps*

An OTC inflation bond pays a real coupon and has a redemption payment of the notional at maturity. It is essentially the same as a government inflation-linked security, with the difference that the credit quality of the issuer now equals that of the swap counterparty. Just like the revenue inflation swap, the inflation leg of an OTC inflation bond can be seen as a portfolio of spot starting zero-coupon inflation swaps. Using the structures discussed earlier, we can replicate it by investing $c$, the real coupon, in a revenue inflation swap with cashflows at the coupon dates and 100 in a zero-coupon inflation swap with a maturity equal to the maturity of the bond. Note that using this structure one can create inflation bonds with different coupon dates/maturities as issued in the cash market. It should be noted that these OTC inflation-linked bonds trade at a premium compared with the OTC inflation-linked bonds with coupon dates/maturities matching the cash market as they are more difficult for investment banks to risk manage due to seasonality risk. OTC inflation bonds are rarely traded as such, but are a useful building block to inflation asset swaps which are discussed in Section 7.4.

## 7.3. Period-on-period inflation swaps

*A period-on-period swap pays periodic inflation*

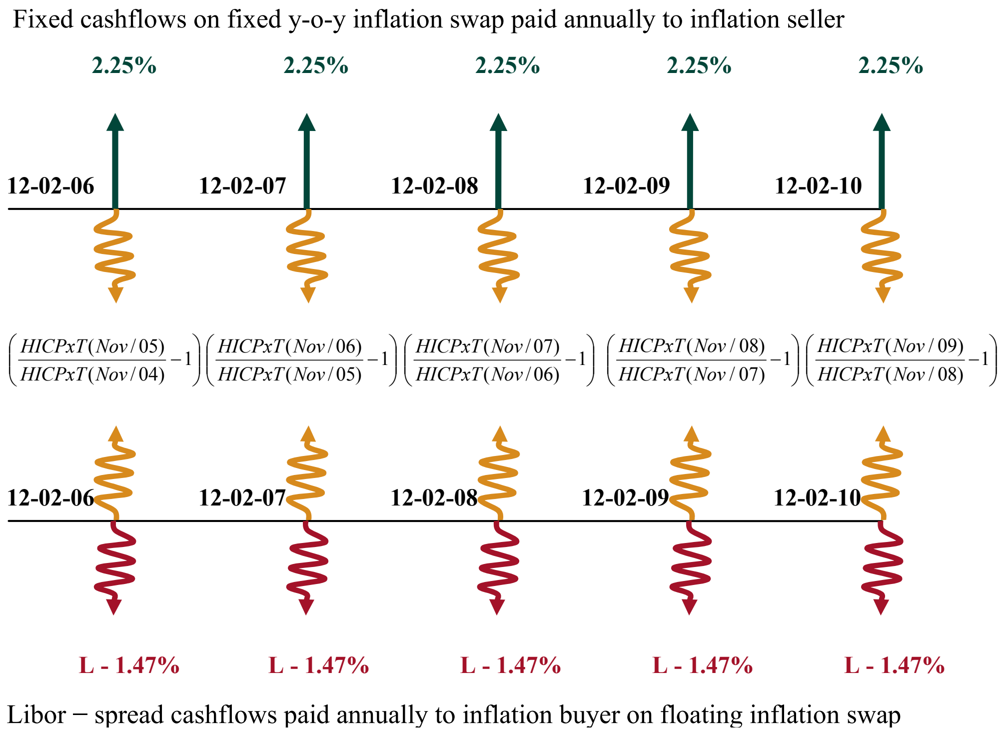
As a revenue inflation swap, a *period-on-period* (p-o-p) *inflation swap* has multiple payments during its life. However, instead of paying the cumulative inflation from the start date up to the coupon dates, it pays the inflation over a number of accrual periods. The most common structure is the year-on-year (y-o-y) inflation swap, which pays annual inflation at the end of each year.  An example term sheet is given in Figure 7.3.

**Figure 7.3. Example of a term sheet for HICPxT year-on-year inflation swap**

| | |
|---|---|
| Notional: | €100,000,000 |
| Index: | HICPxT (non revised) |
| Source: | First publication by Eurostat as shown on Bloomberg CPTFEMU |
| Today: | 10 February 2005 |
| Start date: | 12 February 2005 |
| End date: | 12 February 2010 |
| Rolls: | $12^{\text{th}}$ |
| Payment: | Annual, modified following |
| Day count: | 30/360 unadjusted |
| First fixing: | 115.60 (November 2004) |
| Fixed rate: | 2.25% |
| Fixed leg: | day count fraction × fixed rate |
| Inflation leg: | $\left(\frac{HICPxT(Nov/yy)}{HICPxT(Nov/yy-1)}-1\right)=\left(\frac{I(01-\text{Feb}-yy+1)}{I(01-\text{Feb}-yy)}-1\right)$ for yy = 04,...,09. |

The y-o-y inflation swap in Figure 7.3 is initiated on 12 February 2005 and the inflation payer pays five times annual inflation from November to November every 12 February in the years 2006,…, 2010. An example of a year-on-year inflation swap is shown in Figure 7.4.

**Figure 7.4. Cashflows for 5-year y-o-y inflation swaps (fixed and floating)**




*The cashflows on the inflation leg are given in the middle.*
*Source: Lehman Brothers.*

*The valuation of p-o-p swaps is model-dependent*

The cashflows on the inflation leg can be replicated using a series of forward starting zero-coupon inflation swaps. In the above example we enter into forward starting zero-coupon swaps paying February 2004-2005 inflation,…, February 2008-2009 inflation. Therefore, the valuation can be done in terms of forward starting zero-coupon swaps. We give further details in Appendix A.4.

*Pure period-on-period inflation swaps and annualised period-on-period inflation swaps*

Although the year-on-year inflation swap is the most popular instrument, other period-on-period swaps trade as well. We make a distinction between what we call *pure period-on-period inflation swaps* and *annualised period-on-period inflation swaps*.

A pure p-o-p inflation swap pays the inflation over the period on the inflation leg. For example, a semi-annual pure p-o-p inflation swap with contract months February and August pays the net increase in the index from February to August and the net increase from August to February. As the inflation payments are not on an annual basis, seasonality is an important issue when valuing these swaps.

Just like a y-o-y inflation swap, an annual p-o-p inflation swap pays annual inflation, but it pays it at a higher frequency and weighted with the appropriate day count fraction. For example, a semi-annual annual p-o-p inflation swap with contract months February and August pays half the net index increase from last February to February and half the net index increase from last August to August.[36]

As the period for a year-on-year swap equals a year, a y-o-y inflation swap falls in both categories. Figure 7.5 shows the cashflows for a semi-annual pure and a semi-annual annual inflation swap.

[36] *For convenience, we assumed 30/360 as the basis such that the period between February and August has a day count fraction equal to a half.*

**Figure 7.5. Inflation leg cashflows of 2-year semi-annual period-on-period swaps**

Inflation cashflows of a semi-annual annual p-o-p inflation swap

$$\frac{1}{2}\left(\frac{CPI(Feb/05)}{CPI(Feb/04)}\right) \quad \frac{1}{2}\left(\frac{CPI(Aug/05)}{CPI(Aug/04)}\right) \quad \frac{1}{2}\left(\frac{CPI(Feb/06)}{CPI(Feb/05)}\right) \quad \frac{1}{2}\left(\frac{CPI(Aug/06)}{CPI(Aug/05)}\right)$$

**12-05-05** **12-11-05** **12-05-06** **12-11-06**

Inflation cashflows of a semi-annual pure p-o-p inflation swap

$$\left(\frac{CPI(Feb/05)}{CPI(Aug/04)}\right) \quad \left(\frac{CPI(Aug/05)}{CPI(Feb/05)}\right) \quad \left(\frac{CPI(Feb/06)}{CPI(Aug/05)}\right) \quad \left(\frac{CPI(Aug/06)}{CPI(Feb/06)}\right)$$

**12-05-05** **12-11-05** **12-05-06** **12-11-06**

*The figure plots the inflation leg cashflows from a semi-annual p-o-p inflation swap with annual inflation periods and pure semi-annual inflation swaps. The fixed / floating leg payments are omitted.*
*Source: Lehman Brothers.*

As seasonality plays a role in valuing the pure period-on-period swaps, they would trade at a premium as the inflation seller needs to be rewarded for taking on the seasonal risk. Using high frequency (eg, monthly) annualised period-on-period inflation swaps seems attractive as it spreads inflation payments over the year instead of one lump sum payment each year without a seasonality premium as seasonality does not affect the valuation.

*Marking to market inflation swaps*

As revenue and period-on-period inflation swaps are portfolios of zero-coupon inflation swaps, mark-to-market valuation can be done by marking to market each individual instrument if no specific quotes for the whole structure are available. In the case of inflation versus floating inflation swaps, the positions can be marked to market by marking to market the fixed inflation swap and the interest rate swap individually. See Section 6.1 for an example of marking-to-market a zero-coupon inflation swap.

## 7.4. Inflation asset swaps

*Description*

In general, an asset swap is a synthetic structure recomposing cashflows of existing market instruments. This is usually driven by investors' need for cashflow profiles not attainable in the current market. In this section, we focus on asset swaps of inflation-linked securities and, in particular, inflation-linked bonds. Strictly speaking, asset swaps cannot be classified as inflation derivatives, but they clearly represent a key reference in the valuation of inflation derivatives as they provide a link between the cash and derivatives market.

*An inflation asset swap is the combination of the purchase of an inflation-linked bond and the entry into an inflation swap*

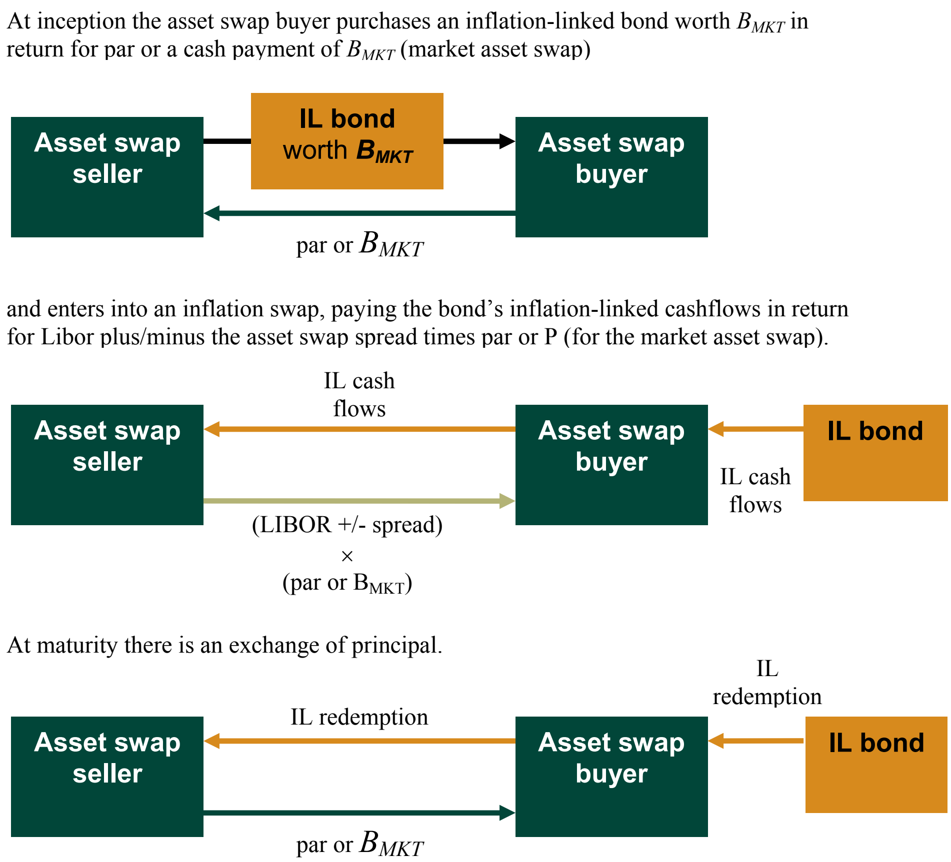
While the interest rate swap market was born in the 1980s, the asset swap market only came to life in the early 1990s. It is most widely used by banks, which use asset swaps to convert their long-term fixed rate assets, typically balance sheet loans and bonds, into floating rate assets in order to match their short-term floating rate liabilities, i.e. depositor accounts. In the past few years the asset swap has become a key structure in the credit markets and is widely used as a reference for credit derivative pricing (see O'Kane, 2001).

Several variations of the inflation asset swap structure exist. In its simplest form it can be treated as consisting of two separate trades. In return for an upfront payment of either par (*par inflation asset swap*) or the dirty price (*market asset swap*), the asset swap buyer:

- Receives an inflation-linked bond from the asset swap seller. Typically the bond is trading away from par. Depending on the passed life of the trade the bond can trade substantially away from par.
- Enters into a series of inflation swaps (equivalent to the OTC inflation bond described in Section 7.1) to pay the asset swap seller inflation coupons equal to that of the asset. In return the asset swap buyer receives regular floating rate payments of Libor plus (or minus) an agreed fixed spread.

The transaction is shown in Figure 7.6. The fixed spread to Libor paid by the asset swap seller is known as the *asset swap spread* and it is set at a breakeven value such that the net present value of the transaction is zero at inception.

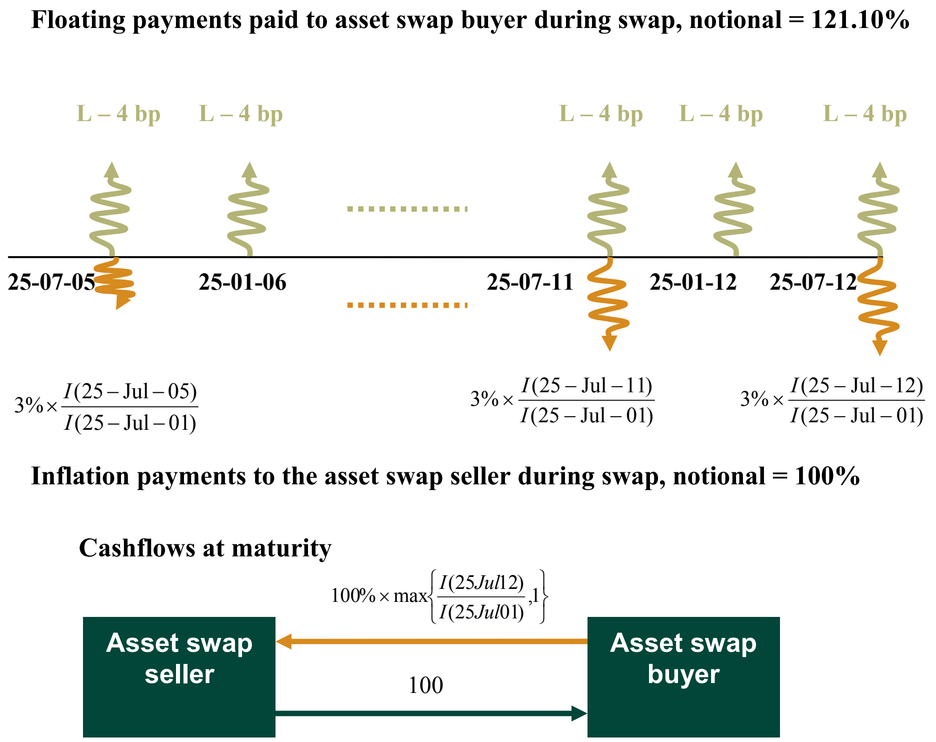
**Figure 7.6. Asset swap mechanics**




*This figure presents the dynamics of an inflation asset swap during the swap and at maturity.*
*Source: Lehman Brothers.*

Figure 7.7 gives an example of a market asset swap of an inflation-linked bond, the OAT€i 2012 issued by France, which has a maturity date of 25 July 2012 and an annual real coupon of 3%. The frequency of the floating leg is taken to be semi-annual. The breakeven value of the asset swap spread makes the net present value of all cashflows equal to zero.

**Figure 7.7. Market asset swap of OAT€i 2012 (3% coupon)**




*Cashflows of an inflation asset swap on the OAT€i 2012. The base reference at 25 July 2001 equalled 108.98710. Source: Lehman Brothers.*

### *Accreting inflation asset swaps*

The inflation asset swaps treated before have the same notional on all the floating payments because the asset swap seller receives the inflation-accreted redemption vs. paying the floating leg notional (par for the par inflation asset swap and the dirty price for the market inflation asset swap). As the inflation-accreted redemption can be substantially bigger than the floating leg notional (either par or the original dirty price), the asset swap seller runs a counterparty risk with respect to the asset swap buyer. For example, for a modest average realised inflation of 2%, the notional accretion over 20 years equals about 48.6% and over 30 years about 81.1%. The usual solution for this counterparty exposure is to put in place a collateralisation agreement through a Credit Support Annex (CSA) agreement. However, this is not possible for all counterparties. One way to mitigate this counterparty risk is to use an accreting inflation asset swap as described below.

*Accreting the floating leg notional mitigates counterparty risk*

Instead of having a constant notional on the floating leg, an accreting inflation asset swap has an accreting notional on the floating leg as well. The accretion can be based on the inflation index, in which case there is no mismatch at maturity. Another possibility is an accretion based on a fixed accretion rate. Unless the realised inflation equals the fixed accretion rate, there will still be a mismatch at maturity, but it is very likely to be much smaller than without an accreting notional. The advantage of the fixed accretion rate is that the floating payments can be easily valued. In the case of the inflation-accreted notional, one has to take into account the dependence between the floating payments and index accretion. As this dependence is fairly hard to estimate, a risk premium will be demanded by the asset swap buyer.

### *Early redemption inflation asset swap*

Another solution to the counterparty risk is to pay the redemption accretion during the life of the swap. The total accretion on the notional is given by:

$$NA(T_N) = \left[\frac{I(T_N)}{I(T_0)} - 1\right] \times \text{Notional},$$

where $NA(T_N)$ equals the redemption accretion from $T_0$ (the start of inflation-linked bond) to $T_N$ which equals maturity. This notional accretion can be split up in $N$ periodical accretions such that:

$$NA(T_N) = \sum_{i=1}^{N} \left[NA(T_i) - NA(T_{i-1})\right],$$

where:

$$NA(T_i) = \left[\frac{I(T_i)}{I(T_0)} - 1\right] \times \text{Notional}.$$

*Using an early redemption swap mitigates counterparty risk without pricing implications*

Due to the early redemption payments of the partial notional accretions at the end of each of the $N$ periods the credit exposure of the asset swap seller is mitigated substantially. In Appendix A.1 we see that this structure has no pricing implications when the payments are discounted using the appropriate discount factor. We show that the early payment at time $T_i$ should equal:

$$\left[NA(T_i) - NA(T_{i-1})\right] D_n(T_i, T_N),$$

that is, the notional increase during the period from $T_{i-1}$ to $T_i$ discounted from maturity to the early payment date, $T_i$.

### *Valuation Aspects*

The valuation of the inflation asset swap strongly depends on the type of asset swap. For the non-accreting swaps, valuation is relatively straightforward and in line with usual asset swaps.

*Calculating the asset swap spread*

The asset swap spread is determined by setting the net present value of all cashflows equal to zero. When discounting cashflows in the swap we use the Libor curve, implying that the parties to the swap have the same credit quality as AA-rated bank counterparties. It is shown in Appendix A.1 that the par asset swap spread is given by:

$$\text{par asset swap spread} = \frac{B_{LIBOR} - B_{MKT}}{PV01}$$

and:

$$\text{market asset swap spread} = \frac{B_{LIBOR} - B_{MKT}}{B_{MKT} \times PV01},$$

where $B_{MKT}$ denotes the value of the inflation-linked bond in the market, $B_{LIBOR}$ is the value of the inflation-linked bond cashflows discounted from the Libor curve, and PV01 stands for the present value of one basis point. See Appendix A.1 for exact definitions. Figure 7.8 gives an example of a market asset swap for the OAT€i 2012.

**Figure 7.8. Cashflows of asset swap on OAT€i 2012 (3% coupon)**

| Date | Libor DF | Index ratio | Inflation cashflow | Nominal rate | Floating cashflow |
|---|---|---|---|---|---|
| 25-Jul-05 | 0.994 | 1.051 | 31,517.92 | 3.15% | -6,511.85 |
| 25-Jan-06 | 0.983 | | | | -13,518.72 |
| 25-Jul-06 | 0.972 | 1.073 | 32,188.74 | 3.22% | -14,349.96 |
| 25-Jan-07 | 0.959 | | | | -16,118.73 |
| 25-Jul-07 | 0.946 | 1.095 | 32,862.20 | 3.29% | -17,169.03 |
| 25-Jan-08 | 0.932 | | | | -18,488.88 |
| 25-Jul-08 | 0.917 | 1.119 | 33,558.02 | 3.36% | -19,256.73 |
| 25-Jan-09 | 0.902 | | | | -20,363.20 |
| 25-Jul-09 | 0.886 | 1.143 | 34,291.19 | 3.43% | -20,887.60 |
| 25-Jan-10 | 0.871 | | | | -22,057.43 |
| 25-Jul-10 | 0.855 | 1.169 | 35,055.23 | 3.51% | -22,513.37 |
| 25-Jan-11 | 0.838 | | | | -23,692.25 |
| 25-Jul-11 | 0.822 | 1.195 | 35,844.87 | 3.58% | -23,999.92 |
| 25-Jan-12 | 0.805 | | | | -25,055.93 |
| 25-Jul-12 | 0.789 | 1.222 | 1,265,816.00 | 125.88% | -1,236,320.92 |
| A | | Bond price | | 121.10 | |
| B | | PV of bond cashflows | | 120.72 | |
| C | | PV of floating leg | | 121.06 | |
| D | | PV01 | | 6.58 | |
| 100×(B-C)/(A×D) | | Asset swap spread | | -4bp | |

*Source: Lehman Brothers.*

The asset swap spreads for asset swaps with accreting floating notionals are harder to compute and are derived in Appendix A.1.

*Applications*

*Asset swaps relieve dealers of balance sheet cost*

The main use of inflation asset swaps is to relieve dealers of balance sheet cost. Inflation dealers provide structured inflation solutions using typically off-balance sheet instruments such as inflation swaps. In order to hedge themselves, they buy inflation-linked bonds which need to be reported on the balance sheet. To relieve them of balance sheet coverage, inflation dealers act as asset swap sellers. By acting as asset swap sellers they get the inflation-linked bonds off the balance sheet while remaining exposed to the inflation-linked payments.

There are several varieties of asset swaps, such as forward asset swaps, cancellable asset swaps and callable asset swaps. However, these asset swap types seem to be more relevant for corporate bonds in credit markets (see O'Kane (2001) for an overview).

## 7.5. Inflation futures

*Description*

*CPI futures market has not really taken off*

The CME (Chicago Mercantile Exchange) started trading futures on the US CPI inflation index on 8 February 2004. The main advantage of CPI futures over zero-coupon inflation swaps is mitigated counterparty risk. The CPI futures traded on the CME are designed to resemble the Eurodollar futures contract. Likely due to the ill-design of the CPI future (the contract traded annualised quarterly inflation), the market so far never really took off. The contract specifications of CPI futures currently traded are given in Figure 7.9.

**Figure 7.9. Contract specification for CPI inflation futures**

| | |
|---|---|
| **Reference Index** | 100 - annualised three-month inflation based on the CPI-U non seasonally adjusted series published monthly by the Bureau of Labor Statistics (BLS). The same index is used for TIPS. |
| **Settlement price** | Final settlement amounts to 100 less the annualised %-change in the CPI-U over the past three months and is rounded to four decimal places. Thus, 100 – 400[CPI($T_i$)/CPI($T_{i-3}$)-1], where $T_i$ denotes the contract month and $T_{i-3}$ the base month. For example, for the June 2003 contract the relevant CPI-U index levels are May 2003 (183.5, released 17 June 2003) and February 2003 (183.1, released 21 March 2003). The final settlement price would have been 100 - 400[183.5/183.1-1]=99.1262. |
| **Contract months** | 12 consecutive March quarterly contract months. |
| **Contract size** | $2,500 times the reference index. |
| **Minimum tick size** | 0.005 index points which amounts to $12.50. |
| **Expiry date** | Trading finishes 7.00am Chicago time on the day the CPI announcement is made in the contract month. In case the announcement is postponed beyond the contract month, trading ceases at 7.00am Chicago time on the first business day following the contract month. |

*Source: CME; Lehman Brothers.*

The CME starts trading inflation futures on the HICPxT index in 2005. One of the main advantages of the Euro contract over the US is that the inflation is annual. The main characteristics of the Euro Consumer Price Index (HICPxT) contract are:

**Figure 7.10. Contract specification for HICPxT inflation futures**

| | |
|---|---|
| **Reference Index** | 100 – annual inflation rate in the 12 month period preceding the contract month based on the Eurozone Harmonised Index of Consumer Prices excluding tobacco published by Eurostat. The same index is used for the French, Italian and Greek Euro inflation-linked bonds. |
| **Settlement price** | Final settlement amounts to 100 less the annual %-change in the HICPxT over the past 12 months and is rounded to four decimal places. Thus: $100-\left[100-\left(\frac{HICPxT(T_i)}{HICPxT(T_{i-12})}-1\right)\right]$, where $T_i$ denotes the contract month and $T_{i-12}$ the base month. For example, for the July 2004 contract the relevant HICPxT index values are June 2004 (115.10, released 16 July 2004) and July 2003 (112.70, released 18 July 2003). The final settlement price would have been: $97.8705=100-\left[100-\left(\frac{115.10}{112.70}-1\right)\right]$. A price of over 100 indicates deflation during the past 12-month period. |
| **Contract months** | 12 consecutive calendar months. |
| **Contract size** | €10,000 times reference index. |
| **Minimum tick size** | 0.01 index points, which amounts to €100.00. |
| **Expiry date** | Trading finishes 4.00pm Greenwich Mean Time (GMT) on the business day preceding the scheduled day the HICPxT announcement is made in the contract month. In case the announcement is postponed beyond the contract month, trading ceases at 4.00pm GMT on the last business day of the contract month. |

*Source: CME; Lehman Brothers.*

*Valuation aspects*

*Due to daily margining, inflation futures have a futures correction*

Although inflation futures closely resemble zero-coupon inflation swaps, their valuation is more complicated. Analogous to interest rate futures contracts, inflation futures have daily margining which, due to the correlation between inflation and interest rates, results in a futures correction. Furthermore, the futures contracts mature once the realised inflation is known, while zero-coupon swaps usually mature with a lag about equal to the lag in the cash market. This early payment of the futures also leads to some pricing adjustments.

*Applications*

**Investing / Trading:**

- Considering their short maturity inflation futures complement the inflation-linked bond markets and allow investors to hedge short-term inflation exposures.
- As the futures trade on 12 consecutive months, investors can take a view on inflation seasonality.

**Hedging:**

- Using strips of inflation futures, one can set up a hedge against bond cashflows.
- As the inflation futures are quoted for consecutive months they can be used to hedge seasonality risk.

## 8. INFLATION VOLATILITY PRODUCTS

### 8.1. Inflation caps and floors

*Description*

Besides swaps, options can also be traded on inflation indices. Caps and floors play a natural role in structures with partial indexation. For instance, a floor on the principal is often included in inflation-linked bonds in order to protect investors against deflation.

Before moving to caps and floors we first look at the simplest of the inflation options. These are calls and puts on zero-coupon swaps. A put option on a zero-coupon swap pays the difference with respect to a (compounded) strike in case inflation turns out to be lower than this pre-specified strike. Combining the put with receiving inflation on a zero-coupon inflation swap results in a structure which pays the maximum of inflation and the strike, thereby flooring the inflation payout at the strike. Hence, puts are usually termed floorlets for interest rates and inflation. An example term sheet of a 1% floor on a zero-coupon swap is given in Figure 8.1.

**Figure 8.1. Example term sheet for HICPxT zero-coupon inflation floorlet**

| | |
|---|---|
| Notional: | €100,000,000 |
| Index: | HICPxT (non revised) |
| Source: | First publication by Eurostat as shown on Bloomberg CPTFEMU |
| Start date: | 12 February 2004 |
| End date: | 12 February 2009 |
| First fixing: | 115.60 (November 2004) |
| Buyer: | $Notional \times \max\left\{(1+1.00\%)^5 - \frac{HICPxT(Nov/09)}{HICPxT(Nov/04)}, 0\right\}$ |
| Seller: | upfront premium |

If the buyer of the floor in the above example is also receiving inflation on a zero-coupon swap, he or she is now protected against an average inflation increase lower than 1%. Floorlets are regularly incorporated in inflation-linked bonds. For instance, all the US TIPS and French OATis have a redemption-protecting floorlet guaranteeing a redemption equal to par.

*Many inflation-linked bonds have embedded zero floors*

In the same way, a call option is commonly referred to as a caplet, because combining selling the call with receiving inflation on a inflation swap results in capping the inflation cashflow on the swap.

The caplets and floorlets can be either spot or forward starting, depending on whether the first fixing of the index is known. Caplets/floorlets on zero-coupon swaps are usually spot starting when traded individually. Forward starting caplets/floorlets are building blocks for caps and floors on period-on-period inflation swaps as we will see.

Caps and floors are usually traded in combination with period-on-period inflation swaps. For example, partial indexation can be achieved by buying a floor and selling a cap combined with receiving inflation in a year-on-year swap. An example term sheet for a 5% inflation cap is shown in Figure 8.2.

**Figure 8.2. Example term sheet for RPI year-on-year inflation floor**

| | |
|---|---|
| Notional: | €100,000,000 |
| Index: | RPI (non revised) |
| Source: | First publication by Eurostat as shown on Bloomberg UKRPI |
| Start date: | 12 May 2004 |
| End date: | 12 May 2009 |
| First fixing: | 183.80 (February 2004) |
| Buyer: | $Notional \times \max\left\{ \frac{HICPxT(Feb/yy)}{HICPxT(Feb/yy-1)} - 5\%, 0 \right\}$ for $yy = 05, \ldots, 09$. |
| Seller: | upfront premium |

For ordinary swaps, an inflation cap-floor parity relation should be satisfied at any time:

*Inflation cap – inflation floor = payer inflation swap,*

where the cap and floor have the same strike. Finally, an inflation collar is the combination of an inflation cap and floor.

*Valuation aspects*

*Standard Black-type formulas can be used for valuation*

Due to the optionality in the product, the valuation of caps and floors requires a model for the dynamics of the index at maturity. For European-style options, the simplest model is a variation on the Black model used for valuing interest rate caps and floors, where instead of forward interest rates, we model the index increase at the option maturity as a random variable which has a lognormal distribution. In Appendix A.5 we provide the valuation formulae.

Because sellers of inflation options typically have to hedge their options dynamically and the bid-offer spreads in the zero-coupon inflation swap market are not yet negligible, transactions costs will be factored into the pricing.

*Applications*

We summarise a number of applications for caps and floors.

**Hedging / Asset Liability Management**

- Caps and floors are particularly well suited for partial indexation schemes.
- A payer of inflation on an inflation swap can limit the uncertain inflation payoff by buying a cap. Yield is sacrificed in exchange for the guarantee that the inflation payments do not exceed a pre-agreed strike.

**Investing / trading**

- Caps and floors can be used to leverage a view on inflation.
- An investor who already has a position in an inflation swap paying inflation can sell a floor for yield enhancement. An investor who is receiving inflation on a inflation swap can sell a cap for yield enhancement.

## 8.2. Inflation swaptions

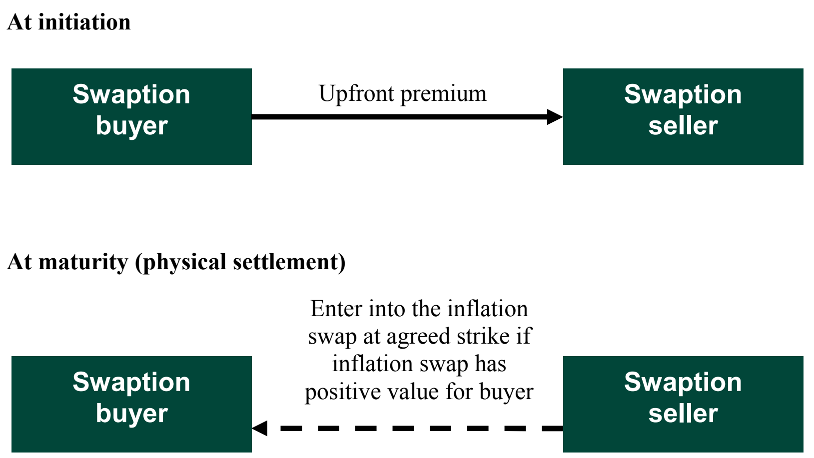
*Description*

*Inflation swaptions can be options on both fixed or floating inflation swaps*

In the same way that the payoff of inflation caps and floors depends on actual fixings of inflation, the payoff of inflation swaptions depends on inflation expectations. In the same way that there are two common types of inflation swaps, there are also two types of inflation swaptions. We denote swaptions on floating inflation swaps as floating inflation swaptions and swaptions on fixed inflation swaps as fixed inflation swaptions. Using swaptions we can create callable and cancellable inflation swaps in the same manner as for interest rate swaps.

A payer/receiver *inflation swaption* gives the right to enter into a payer/receiver inflation swap with tenor $T_N$-$T_0$ at time $T_0$ at a pre-specified coupon $\kappa$. We denote this as a $T_0 \times (T_N - T_0)$ inflation swaption. The mechanics of an inflation swaption are given in Figure 8.3

**Figure 8.3. Mechanics of a (physically settled) swaption on an inflation swap**




*Source: Lehman Brothers.*

We start by introducing the fixed inflation swaptions. An example term sheet is given in Figure 8.4.

**Figure 8.4. Example term sheet fixed payer inflation swaption**

| | |
|---|---|
| Notional: | €100,000,000 |
| Index: | HICPxT (non revised) |
| Source: | First publication by Eurostat as shown on Bloomberg CPTFEMU |
| Start date: | 12 May 2005 |
| Option end date: | 12 May 2006 |
| Swap end date: | 12 May 2011 |
| Type: | Zero-coupon |
| Reference month: | February |
| First fixing: | not yet known. |
| Buyer: | The right to enter into a fixed payer inflation swap starting 12 May 2005 and ending 12 May 2011 with payoff $(1+b)^5 - \frac{HICPxT(Feb/11)}{HICPxT(Feb/06)}$, where $b$ denotes the inflation swap rate at 12 May 2006 for the February 2006 to February 2011 inflation swap. |
| Seller: | upfront premium |

The above fixed payer inflation swaption gives the buyer the right to enter at 12 May 2006 into a fixed payer zero-coupon inflation swap ending 12 May 2011. This inflation swap pays the difference between the compounded market inflation rate at 12 May 2006 minus the ratio of the HICPxT index values in February 2011 and February 2006.

Besides zero-coupon swaptions, period-on-period inflation swaps are also possible. In the next example term sheet we describe a floating receiver year-on-year inflation swap, also called a real rate swaption (Figure 8.5).

**Figure 8.5. Example term sheet floating receiver inflation swaption**

| | |
|---|---|
| Notional: | €100,000,000 |
| Index: | HICPxT (non revised) |
| Source: | First publication by Eurostat as shown on Bloomberg CPTFEMU |
| Start date: | 12 May 2004 |
| Option end date: | 12 May 2006 |
| Swap end date: | 12 May 2011 |
| Type: | Year-on-year |
| Floating rate: | 3M Euribor – 2.5%, quarterly paid (ACT/360) in Feb, May, Aug, Nov. |
| Reference month: | February |
| First fixing: | Not yet known. |
| Buyer: | The right to enter into a floating receiver inflation swap starting 12-May-06 and ending 12-May-11 with cash flows $Notional \times \left( \frac{HICPxT(Feb/yy)}{HICPxT(Feb/yy-1)} - 1 \right)$ for $yy = 07, \ldots, 11$, and -*Notional*×(EURIBOR-2.50%)×*daycount* starting 12 Aug 2006 and ending 12 May 2011. |
| Seller: | upfront premium |

The above floating receiver inflation swaption gives the buyer the right to enter at 12 May 2006 into a floating receiver year-on-year inflation swap ending 12 May 2011. This inflation swap pays the annual inflation every year from February to February in return for quarterly payments of Euribor – 2.50% (the agreed spread).

*Valuation aspects*

*Variants of the Black model can be used for pricing*

Due to the optionality in the product, the valuation of fixed inflation swaptions requires a model for the dynamics of the breakeven rate on the inflation swap. As a floating inflation swap is a portfolio of a fixed inflation swap and an interest rate swap we need to model the dynamics of the breakeven rate on the fixed inflation swap, the nominal swap rate, and their interdependence for valuation of the floating inflation swaption. For European-style options, the simplest models are again variations on the Black model used for valuation, where instead of forward interest rates we model the forward swap rate for the fixed inflation swaption as a lognormal random variable. The floating inflation swaption can be valued modelling the spread between the nominal swap rate and the breakeven inflation rate as a lognormal random variable. In Appendix A.7 we provide the valuation formulae.

As sellers of inflation options typically have to hedge their options dynamically and the bid-offer spreads in the zero-coupon inflation swap market are not yet negligible, transactions costs will be factored into the pricing just as for caps and floors.

*Applications*

**Hedging / Asset Liability Management**

- Inflation swaptions give investors the right to exchange future inflation-linked cash flows for fixed or floating payments. For instance, investors know that they will receive inflation-linked cashflows for five years starting in six months' time. In exchange for an upfront payment, they receive the right to enter an inflation swap at a currently agreed strike value.

**Investing / trading**

Inflation swaptions are well suited to leverage a view on inflation.

- If investors already have a position in an inflation swap, they can enter into an inflation swaption on the opposite swap in order to have the right to cancel the inflation swap.
- Floating inflation swaptions allow investors to leverage a view on the spread between the nominal swap rate and the breakeven inflation rate.

### 8.3. LPI swaps

*LPI swaps are mainly traded on RPI*

LPI (Limited Price Index) swaps are useful instruments for institutions that have limited indexation schemes. They are especially useful for the UK market, which has over £200bn of LPI liabilities. LPI swaps come in a variety of flavours, but what they have in common is that, in one way or another, the inflation payment is capped and/or floored. If the inflation payment is both capped and floored, we call it collared. We describe two structures that are related to the UK pension regulation, but others exist as well.

*Zero-coupon LPI swap*

The zero-coupon LPI swap is particularly useful for pension funds that have liabilities related to the limited indexation of pensions in deferment as introduced in the 1995 Pension Act. They have liabilities of the following form:

$$L(T) = L(0) \times \max\left\{\min\left\{\frac{RPI(T)}{RPI(0)}, 1.05^T\right\}, 1.0\right\},$$

where $L(0)$ denotes the liability when the retiree retires.

**Figure 8.6. Example term sheet for zero-coupon LPI swap**

| | |
|---|---|
| Notional: | £100,000,000 |
| Index: | RPI (non revised) |
| Source: | First publication by Eurostat as shown on Bloomberg UKRPI |
| Today: | 29-Jan-05 |
| Start date: | 01-Feb-05 |
| End date: | 01-Feb-10 |
| Rolls: | $1^{st}$ |
| Payment: | Annual, modified following |
| Day count: | 30/360 unadjusted |
| First fixing: | $LPI(0) = 1.0$ |
| Fixed leg: | $(1+2.89\%)^5$ |
| Inflation leg: | $LPI(T_e) = LPI(T_s) \times \max\left\{\min\left\{\frac{I(T_e)}{I(T_s)}, 1.05^{T_e - T_s}\right\} 1.0\right\}$ for $T_s$ = 01-Dec-04 and $T_e$=01-Dec-09. |

The zero-coupon LPI swap can be valued using a portfolio of a zero-coupon inflation swap, a zero-coupon inflation call, and a zero-coupon inflation put. Of course, other strike values for the cap and floor can be used as well. The 5% and 0% used are a result of the 1995 Pension Act legislation.

*Periodic LPI swaps*

A periodic LPI swap has periodic, typically annual, payments similar to a revenue inflation swap with the exception that the payoff is defined in terms of an artificially created index, the LPI, instead of the ordinary index. The LPI can be defined as:

$$LPI(T_i) = LPI(T_{i-1}) \times \max\left(\min\left(\frac{I(T_i)}{I(T_{i-1})}, 1.05^{T_i - T_{i-1}}\right), 1\right) \text{ for } i = 1,\ldots, N,$$

with $LPI(T_0)=I(T_0)$, the value of an inflation index (e.g. RPI) at $T_0$. The LPI is capped at an annual growth of 5% and floored at 0%. Again, other cap and floor strike values can be used. In Figure 8.7 we present an example term sheet.

**Figure 8.7. Example term sheet for annual LPI swap**

| | |
|---|---|
| Notional: | £100,000,000 |
| Index: | RPI (non revised) |
| Source: | First publication by Eurostat as shown on Bloomberg UKRPI |
| Today: | 29 January 2005 |
| Start date: | 1 February 2005 |
| End date: | 1 December 2010 |
| Rolls: | $1^{st}$ |
| Payment: | Annual, modified following |
| Day count: | 30/360 unadjusted |
| Floating leg: | (GBP) Libor + 0.20% |
| Inflation leg: | $b \times LPI(i)$ for $i$ = 1,…,5 with $LPI(0)$ = 1.0 and |
| | $LPI(i) = LPI(i-1) \times \max\left(\min\left(\frac{RPI(Dec/yy)}{RPI(Dec/yy-1)}, 1.05\right), 1\right)$ for $i = 1,\ldots,5.$ |
| | Furthermore, a redemption pick-up is paid at maturity equal to |
| | (LPI(5)-1)×Notional. |

*Valuation aspects*

Unlike the zero-coupon LPI structure, the period-on-period LPI swap cannot be valued using an appropriate static portfolio inflation swaps, caps, and floors. In order to value an LPI swap one needs a proper inflation term structure model. An LPI swap can then be valued by simulation of the term structure model.

## 8.4. Inflation spread options

*Description*

Inflation spread options pay the spread between two inflation indices, for example, harmonised European inflation and Dutch inflation, if positive. These products can be useful for pension funds in countries where no inflation-linked bonds are issued with respect to their benchmark inflation index. For instance, let's assume a pension fund is interested in being benchmarked against index A (e.g. Dutch HICP), but the liquid index (e.g. Euro HICPxT) is index B. A typical payoff to the holder is given by:

$$\max\{\text{index increase index A} - \text{index increase index B}, 0\}$$

The seller gets an upfront payment in return. Combining a spread option with a simple inflation paying instrument on the liquid index, B, gives a "best of two inflation indices" instrument.

$$\max\{\text{index increase index A}, \text{index increase index B}\}$$

*Valuation aspects*

*A simple extension of the Black model can be used for pricing*

The optionality in the product makes the valuation of spread options model-dependent. Valuing the spread options requires modelling the spread or the joint dynamics of the indices. European-style options can be valued using Margrabe's (1978) formula, which is an extension of the Black model. We present the valuation formula in Appendix A.6.

Furthermore, the dealers take on a risk with respect to a non-traded index which they can hardly hedge. In return they will demand a risk premium.

*Applications*

**Hedging / Asset Liability Management**

- Inflation spread options provide a neat hedge against the risk that the index used for hedging inflation exposures (usually a liquid index, such as HICPxT) diverges from the benchmark index for the investor.

**Investing / trading**

- Inflation spread options allow investors to express a view about spread volatility separate from a view about the direction of the index spread.

## 9. STRUCTURED INFLATION PRODUCTS

Currently, investors have the option to get inflation-linked returns on bonds via inflation-linked bonds. However, natural receivers of inflation such as pension funds and insurance companies have portfolios consisting of bonds, equity, credit-linked products, an so on. In addition to being able to get real bond-like returns, they could be interested in investing in products which generate real equity or real credit-linked returns.

The growth of pure inflation derivatives, as described in the previous section, has opened the door for structured hybrid products linked to inflation. In this section, we consider a selection of inflation-linked structures.

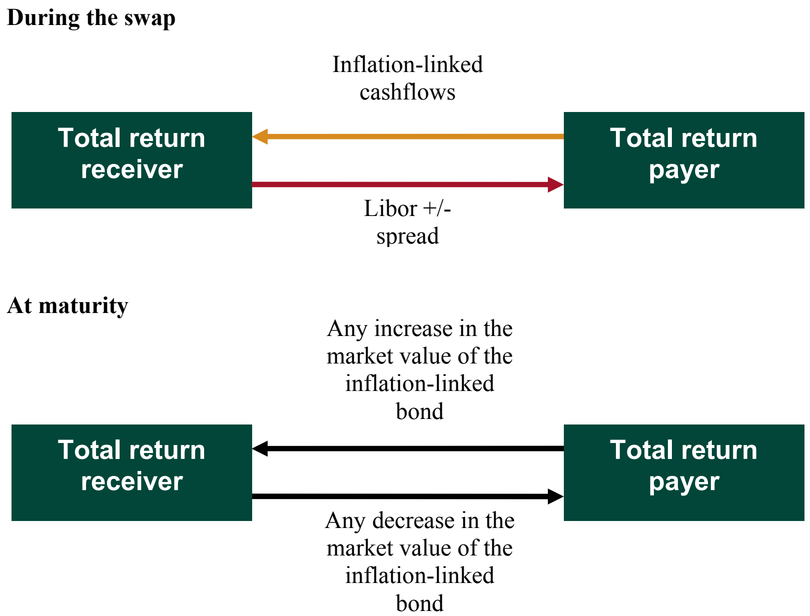
### 9.1. Total return swaps

*Description*

*A total return swap is a tool for regulation and balance sheet arbitrage rather than investing*

A *total return swap* (TRS) is a contract that allows investors to receive all the cashflow benefits of owning an asset without actually holding the physical asset on their balance sheets. As such, a total return swap is more a tool for balance sheet arbitrage or regulation arbitrage than for investment purposes. First, we discuss total return swaps for the case where the asset is an inflation-linked bond. Figure 9.1 shows the mechanics.

**Figure 9.1. Mechanics of a total return swap**




*Source: Lehman Brothers.*

At trade inception, the total return receiver agrees to make payments of Libor plus or minus a fixed spread to the total return payer, in return for the coupons paid by some inflation-linked security. At the end of the term of the total return swap, the total return payer must then pay the difference between the final market price of the asset and the initial price of the asset, if positive. If the difference between the final market price of the asset and the initial price is negative, the total return receiver needs to pay the difference to the total return payer.

*Total return swaps can be linked to individual securities or indices*

Total return swaps do not necessarily have to be linked to a particular security. For example, one may wish to link the total return to an index such as the Lehman Brothers Global Inflation-Linked Aggregate Index, thereby creating what is known as an *index swap*. Lehman Brothers has several inflation-related indices, and we present the most important ones in Figure 9.2.

**Figure 9.2. Lehman inflation indices**

| Lehman indices | Description |
|---|---|
| Global inflation-linked | Includes all inflation-linked bonds by the US, the UK, Canada, Sweden, Italy, France, and Greece. |
| Euro-zone | Includes all inflation-linked bond linked to the European HICPxT index. |
| Pan-Euro | Includes all inflation-linked bonds excluding US TIPS and Canadian RRBs. |
| US TIPS | Includes all US inflation-linked bonds, TIPS |

*In addition to the indices above, country-specific indices for Canada, UK, Sweden, France (including both French and Euro indices), Italy, and Greece also exist.*
*Source: Lehman Brothers.*

### *Valuation aspects*

*Pricing is determined by the cost of funding the hedge position*

A total return payer can statically hedge the total return swap by buying the inflation-linked security, funding it on balance sheet, and selling it at trade maturity. Indeed, one way the holder of an asset can hedge against changes in the value of an asset is to become a payer on a total return swap. This means that the cost of the trade, i.e. the spread, will mainly depend on the funding cost of the total return payer. When the total return swap is made with respect to an inflation index, the total return payer will have to hedge by buying the index. The cost of this will be represented by the cost of buying the individual bonds. However, typically the bid-offer will be lower than replicating the individual bonds. This is because the index swap dealer will be more willing to take outright market risks and basis risks than the investor. Also, by having a reasonably balanced book in index swaps, the dealer will be able to aggregate risks and so reduce hedging costs. For inflation indices, the difference will be reasonably small as the indices consist of fairly few bonds. As the indices grow in constituents, the replicating portfolio becomes more expensive compared with the index in terms of bid-offer spreads.

### *Applications*

An investor may be interested in a total return swap on an inflation-linked security for several reasons.

**Funding / Leverage**

- As an unfunded transaction, total return swaps make it easy to leverage a view on inflation either in individual assets or a portfolio of inflation-linked bonds.
- They enable investors to obtain off-balance-sheet exposure to assets to which they might otherwise be precluded for tax, political or other reasons.
- Buying and selling index swaps may be more liquid than trading the underlying assets. Bid-offer spreads will usually be tighter.

**Trading / Investing**

- Total return swaps allow a total return payer to short an asset without actually selling it. This might be useful from the point of view of temporarily hedging an inflation exposure. Another reason might be an expected underperformance in the short run, while maintaining confidence in its long-run performance.
- Using a total return swap, one can create a synthetic structure with the required maturity.
- Clients can use the index swap to benchmark their portfolios to standard inflation indices, such as those summarised in Figure 9.2.
- A portfolio manager can replicate an index without incurring a tracking error.

## 9.2. Inflation-linked equity

*Description*

Inflation-linked equity is the equity with a fixed maturity equivalent to an inflation-linked bond with a fixed maturity. For example, consider the situation where a pension fund has inflation-linked liabilities and a substantial equity and bond portfolio (we discuss CDOs in the next section). The pension fund can buy inflation-linked bonds or enter into zero-coupon swaps to match its bond portfolio to the inflation-linked liabilities. Inflation-linked equity allows the pension fund to link the equity to inflation as well. The value of real equity at maturity $T$ is given by:

$$R(T) = I(T)S(T),$$

where $S(T)$ denotes the value of the equity portfolio at time $T$. The idea behind real equity is similar to that of real bonds which pay $I(T)D_r(T, T)=I(T)$ at time $T$.

*Valuation aspects*

*Inflation-linked equity is sensitive to the inflation-equity correlation*

The payoff of inflation-linked equity depends on co-movement between the inflation index increase and the equity increase. This co-movement makes the valuation model-dependent. One model is given by assuming that the inflation index and the equity are both lognormal distributed with a deterministic correlation. The value is then given by:

$$R(0) = \frac{I(0,T)S(0)}{D_n(0,T)} \exp\left( \int_0^T \rho_{IS}(u)\sigma_I(u,T)\sigma_S(u,T)du \right),$$

where $\sigma_I(u,T)$ denotes the instantaneous volatility of the breakeven reference number $I(0,T)$ today, $\sigma_S(u,T)$ denotes the instantaneous volatility of the forward equity price, that is, $S(0) / D_n(0,T)$, and $\rho_{IS}(u)$ denotes their instantaneous correlation.

*Applications*

As for inflation-linked bonds, the main application of inflation-linked equity is in the ALM context. We summarise some potential uses.

**Hedging / ALM**

- Inflation-linked equity allows investors to hedge their inflation-linked liabilities while enjoying equity-like returns.

**Investing / Trading**

- Inflation-linked equity allows investors to take a view on the dependence between inflation and equity returns.

## 9.3. Inflation-linked equity options

*Description*

A popular structure for investors in nominal space to hedge their downside is to combine a stock portfolio and a (typically ATM) put. This structure ensures that the value of an investor's portfolio at maturity equals at least its current value. This payoff is given by:

$$S(T) + \max\left(S(0) - S(T), 0\right) = S(0)\max\left(1, 1 + r^s(0,T)\right),$$

where $r^S(0,T)=S(T)/S(0)-1$ denotes the gross return on the equity portfolio. In this structure, the nominal value of the equity portfolio remains at least equal to its original nominal value. With inflation-linked liabilities, the real value of the equity portfolio decreases if the benchmark index rises. Using an inflation-linked equity option, we can construct a structure which guarantees that the real value of the equity portfolio remains at least equal to the original real value. This can be constructed in the following manner:

*Inflation-linked equity options allow structures that guarantee the real value of equity*

$$S(t)+S(0)\max\big(i(0,T),r^{s}(0,T)\big)=S(T)+S(0)\max\big(i(0,T)-r^{s}(0,T),0\big).$$

The structure can be replicated by investing in the stock and buying a spread option which pays off the difference between inflation and the equity return if positive.

*Valuation aspects*

As for inflation spread options, inflation-linked equity options require modelling the spread or the joint dynamics of the indices. European-style options can be valued using Margrabe's (1978) formula, which is an extension of the Black model.

*Applications*

Inflation-linked equity options have a number of applications which are summarised below.

**Hedging / Asset Liability Management**

- Inflation-linked equity options allow investors to guarantee the real value in a stock and put construction.

**Investing / Trading**

- Inflation-linked equity options allow investors to leverage a view on equity and inflation correlation.

## 9.4. Inflation-linked credit default swaps

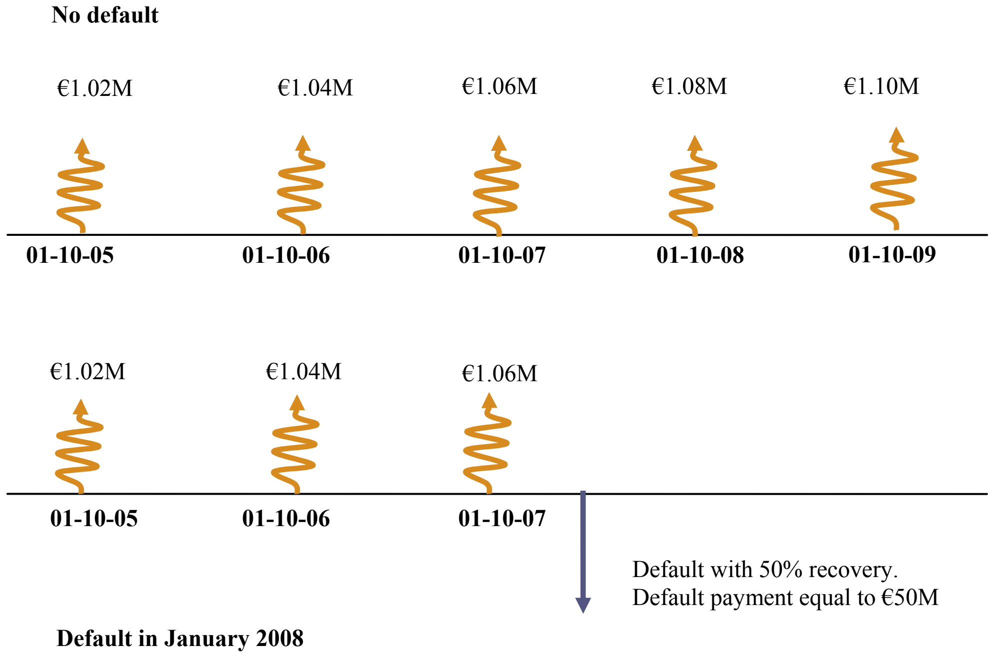
*Description*

A credit default swap (CDS) is a bilateral contract that enables an investor to buy protection against the default of an asset issued by a specified reference entity. In the case of a defined credit event (typically, default), the protection buyer receives a default payment intended to compensate against the loss in investment. In return the inflation buyer pays a fee. This fee may be up-front or in the form of a regular series of cashflows.

*The CDS spread is inflation-linked*

In principle, one can envisage several types of inflation-linked credit default swaps. We focus on a CDS on which the protection buyer pays a real spread. The default payment is as usual. We consider an example trade: a €100 million, 5-year inflation-linked CDS. The real spread equals 1% annually. For convenience, we assume that inflation will equal 2% in all subsequent years to compute projected cashflows. The premium cashflows are then given by $1.02^{i}\times$€ 1M for year $i=1,\ldots,5$ if no default has occurred. If a default occurs, the premium cashflows cancel out. We present some example cashflows to illustrate default and no-default scenarios in Figure 9.3.

**Figure 9.3. Example cashflows of 5-year inflation-linked CDS**




*Source: Lehman Brothers.*

### *Valuation aspects*

*The value depends on the inflation and default correlation*

An inflation-linked CDS pays a real spread whereas an ordinary CDS pays a fixed spread. If we knew which spread payment would occur we could easily transform the fixed premium cashflows to inflation-linked premium cashflows using an inflation swap. However, due to the uncertain credit event, this is not possible. Using a curve of survival probabilities (which gives the probability that the asset has survived to that point in time), we can compute the expected inflation-linked cashflows. When defaults are uncorrelated with inflation, the value of an individual payment is the discounted value of the expected notional times the expected index ratio at the time of the payment.[37] In the case of positive correlation between inflation and the credit event, the premium will be lower than when there is no correlation. In the case of negative correlation, the value will be higher. The intuition behind this is that in periods of high inflation the premium is likely to be paid longer in the case of negative correlation and vice versa.

### *Applications*

**Hedging / ALM**

- An inflation-linked CDS provides a natural way for investors with inflation-linked liabilities to sell protection on a CDS. Unlike with fixed or nominal floating spread payment, investors receive inflation-linked spread payments to match their liabilities.

**Investing / Trading**

- An inflation-linked CDS allows investors to leverage a view on credit and inflation correlation.

[37] *The expectation is taken with respect to the appropriate pricing measure.*

## 9.5. Inflation-linked CDO

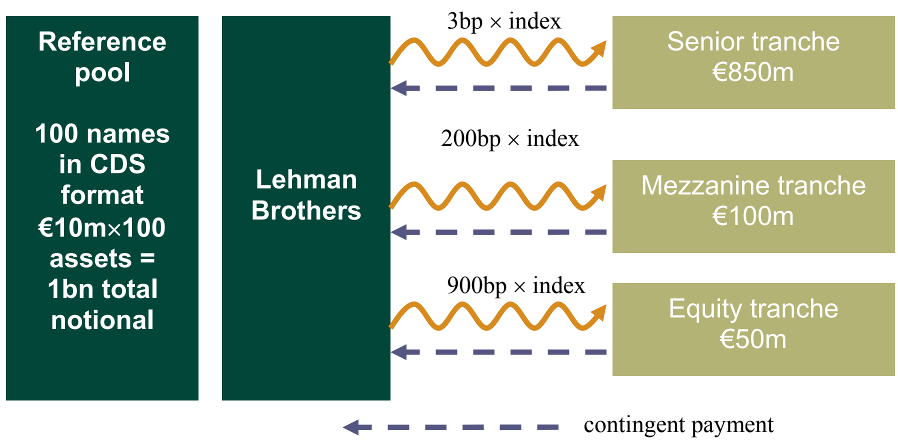
*Description*

A collateralised debt obligation (CDO) is a structure of credit risky securities whose cashflows are linked to the incidence of default in a pool of debt instruments. These debts can be, for example, loans, lines of credit, asset-backed securities, corporate and sovereign bonds, and other CDOs (typically denoted $CDO^2$). For more detailed information on CDO and other credit derivatives, see O'Kane (2001) and O'Kane *et al.* (2004).

*The CDO spread is inflation-linked*

The performance of an inflation-linked synthetic CDO is linked to the incidence of default in a portfolio of CDS and inflation swaps. The CDO redistributes the credit risk by allowing different tranches to take these default losses in a specific order. This order is called the waterfall. To see how the CDO redistributes the credit risk, we consider the example in Figure 9.4.

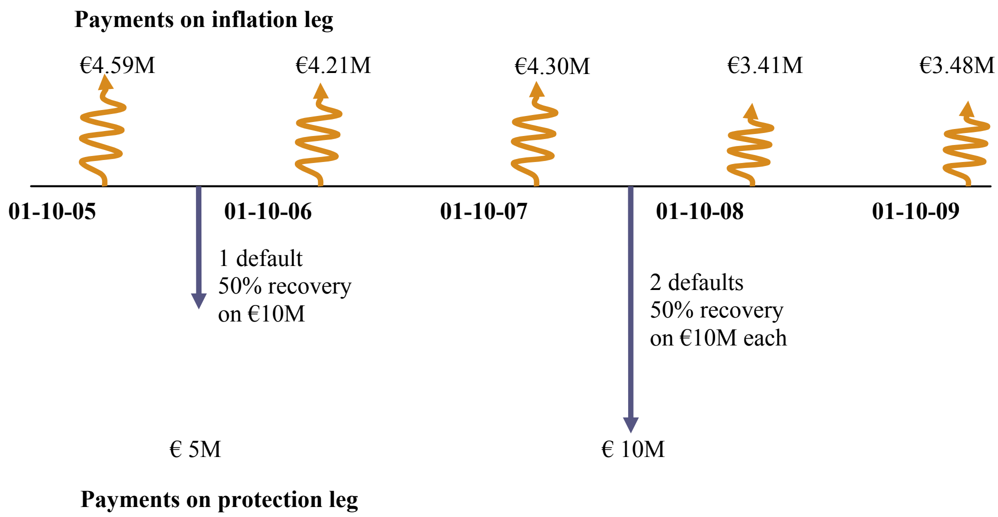
**Figure 9.4. Structure of an inflation-linked CDO**




*Source: Lehman Brothers.*

This CDO is based on a reference pool of 100 CDS, each with €10m notional. The credit risk is redistributed in three tranches. The equity tranche assumes the first €50m of losses. As it is the riskiest tranche, it is paid the highest spread. Next we have the mezzanine tranche which assumes the next €100m of losses. It is less risky than the equity tranche, and therefore pays a lower spread. Finally, we have the senior tranche which is protected by €150m of subordination. To get an idea of the risk of the senior tranche, we note that it would require more than 25% of the assets to default with a recovery of 40% before the senior tranche takes a principal loss. Consequently, it is paid an even lower spread. All spreads are inflation-linked in the same manner; the premium received being the fixed real spread times the inflation index with the premium paid on the outstanding notional. In Figure 9.5 we give an example of the payments of an equity tranche of a 5-year inflation-linked CDO, which initiates on 1 October 2004.

**Figure 9.5. Example cashflows of 5-year inflation-linked CDO equity tranche**




*Due to inflation the protection can become more expensive over the life over the CDO*

We assume that over the life of the trade inflation equals 2%. The equity tranche in the above example is paid an inflation-linked spread of 900bp. For the first payment this equals 9% times €50m times 1.02 (coming from the 2% inflation). In year 2 we see a default with a recovery of 50% which reduces the notional to €45m resulting in a floating payment of €45m times 9% times $1.02^2$ (cumulative inflation up to year 2) and a default payment from the equity tranche holder of €5m. No defaults in year 3 lets the floating payment grow with inflation. Due to two 50% recovery defaults in year 4 the notional on the floating leg decreases to €35m (the floating payment now equals $1.02^4 \times 9\% \times$€35m=€3.41m) and the equity tranche holder needs to pay a €10m default payment. In the last year the floating payment grows with inflation.

*Valuation aspects*

*The value depends on the inflation and default correlation*

Whereas a normal CDO pays a fixed nominal spread, an inflation-linked CDO pays a real spread times an inflation index. If the notionals for the payments were known, the valuation could be simply accomplished using fixed inflation swaps with the appropriate notionals. By the nature of the CDO, the notionals are unknown and the correlation of the defaults and inflation becomes an issue for valuation. When defaults are uncorrelated with inflation, the value of an individual payment is the discounted value of the expected notional times the expected index ratio at the time of the payment.[38] In the case of positive correlation between inflation and defaults, the value will be lower than if there is no correlation. In the case of negative correlation, the value will be higher. The intuition behind this is that in periods of high inflation the expected notionals are high in the case of negative correlation and vice versa.

*Applications*

**Hedging / ALM**

- Inflation-linked CDOs provide a natural way for investors with inflation-linked liabilities to invest in CDOs. They receive the yield pick-up of CDOs while receiving inflation-linked cashflows to match the liabilities.

**Investing / Trading**

- Inflation-linked CDOs allow investors to leverage a view on credit and inflation correlation.

[38] *The expectation is taken with respect to the appropriate pricing measure.*

## 10. LEGAL, REGULATORY, AND ACCOUNTING ISSUES

### 10.1. ISDA inflation derivatives documentation

The ISDA recently published documentation on inflation definitions supplementing the "ISDA Master Agreements".[39] The main issues relate to delay and disruption in the publication of the inflation index. Furthermore, it defines the most relevant indices.

#### *10.1.1 Delay of publication*

*A substitute index is used if the reference index is not published on time*

If the reference index has not been published five business days prior to the next payment date for the transaction related to that index, the calculation agent shall use a substitute index level using the following methodology:

1. If applicable, the calculation agent takes the same action to determine the substitute index level as that specified in the terms and conditions of the related bond. The related bond, if any, is specified in the confirmation of the trade. A related bond is typically specified for asset swaps, but not for inflation swaps.
2. If 1. does not result in a substitute index level for the affected payment date the calculation agent determines the substitute index level as follows:

$$\textit{substitute index level} = \textit{Base level} \times (\textit{Latest level} \,/\, \textit{Reference level})$$

where
*Base level* means the level of the index 12 calendar months prior to the month for which the substitute index level (definitive or provisional) is being determined, for example, December 2005.
*Latest level* means the latest available level (definitive or provisional) for the index, for example, November 2005.
*Reference level* means the level (definitive or provisional) of the index 12 calendar months prior to the month to which the latest level is referring, for example, November 2004.

If a relevant level is published after five business days prior to the next payment date, no adjustments will be made to the transaction. The determined substitute reference level will be the definitive level for that reference month.

#### *10.1.2 Successor index*

*A replacement index will be used if the index no longer gets published*

If, during the term of the transaction, the index sponsor announces that an index will no longer be published or announced but will be superseded by a replacement index specified by the index sponsor, and the calculation agent determines that the replacement index is calculated using the same or similar methodology as the original index, this index is deemed the successor index.

#### *10.1.3 Cessation of publication*

If an index has not been published for two consecutive months or if the index sponsor (publisher) has announced that it will no longer publish the index, the calculation agent shall determine a successor index for the purpose of the transaction using the following methodology:

1. If a successor index has been designated by the calculation agent pursuant to the terms and conditions of the related bond, such successor index shall be designated a successor index hereunder.
2. If no related bond exists, the calculation agent shall ask five leading independent dealers to state what the replacement index shall be. If three or more dealers out of at

[39] *See "2005 ISDA Inflation Derivatives Definitions" on www.isda.org.*

least four responses state the same index, this index will be deemed the successor index. If, out of three responses, two or more dealers state the same index, this index will be deemed the successor index. If no successor index has been decided following responses from dealers by the third business day prior to the next payment date or by the date that is five business days after the last payment date (if no further payment dates are scheduled), the calculation agent determines an appropriate alternative index. This alternative index will be deemed the successor index. If the calculation agent determines that there is no appropriate alternative index, a termination event occurs and both parties are affected parties as defined in the 2002 ISDA Master Agreement.

#### *10.1.4 Rebasing the index*

*If an index is rebased the rebased index will be used going forward*

If the calculation agent determines that the index has been or will be rebased at any time, the rebased index will be used from then on. However, the calculation agent shall make adjustments pursuant to the terms and conditions of the related bond, if any. If there is no related bond, the calculation agent shall make adjustments to the past levels of the rebased index so that rebased index levels prior to the rebase date reflect the same inflation rate as before it was rebased.

#### *10.1.5 Material modification prior to payment date*

If prior to five business days before a payment date an index sponsor announces that it will make a material change to an index, then the calculation agent shall make any adjustments to the index consistent with adjustments made to the related bond. If there is no related bond, only those adjustments necessary for the modified index to continue as the index will be made.

#### *10.1.6 Manifest error in publication*

If, within 30 days of publication, the calculation agent is notified that the index level has to be corrected to remedy a material error in its original publication, the calculation agent will notify the parties of the correction and the amount payable as a result of that correction.

### 10.2. Regulation for pension funds

*Stronger indexation requirements for pension funds will probably boost the inflation market*

An important role in the development of the inflation derivatives market (especially in Europe) will be played by the regulatory frameworks for pension funds in their respective countries. For instance, in the Netherlands, which probably has the most developed European pension industry, the social partners (employers and employees) decided that most pension funds need to provide only conditional indexation. Conditional indexation means that pension funds only need to provide indexation if they have sufficient funds to allow this. Unconditional indexation is a stronger requirement, in which a pension fund guarantees an inflation-adjusted pension. Unconditional indexation schemes would likely have a strong positive impact on the inflation market as demand for inflation products by pension funds would almost certainly increase. So far, no major entities have adopted an unconditional indexation scheme.

### 10.3. Accounting standards

*European companies need to follow the new International Accounting Standards*

In 2005 the European Union adopted the accounting standards as set by the International Accounting Standards Board (IASB). The introduction of these accounting standards obliges all European (including the UK) public companies listed on any exchange in the European Union to prepare their financial accounts following the International Financial Reporting Standards (IFRS). In the derivatives market, all companies are affected via IAS-39, the IFRS standard covering derivatives. The US equivalent to IAS-39, FAS-133, which was introduced several years ago, has more or less similar implications for inflation derivatives by US entities.

IAS-39 requires all derivatives to be valued on a mark-to-market basis, which essentially means that they are transferred to the balance sheet. Changes in the mark-to-market valuation will therefore affect profit and loss account of companies, which likely increases volatility in companies' earnings statements. Only if a company can demonstrate that the derivative is used for hedge accounting purposes, will it need not be value the derivative security on a mark-to-market basis. For inflation derivatives, this means that companies with inflation-linked cashflows need to demonstrate that an inflation swap is an effective hedge for its inflation-linked cashflows. If companies have cashflows indirectly linked to inflation (e.g. supermarkets) this can be problematic and likely costly.

In the inflation derivatives market all companies with defined benefit contributions will also be affected by IAS-19, which prescribes the accounting and disclosure for employee benefits (that is, all forms of consideration given by an enterprise in exchange for service rendered by employees). The principle underlying all of the detailed requirements of the Standard is that the cost of providing employee benefits should be recognised in the period in which the benefit is earned by the employee, rather than when it is paid or payable. Therefore, companies are required to mark-to-market the value of indexation schemes.

In the UK the national Accounting Standards Board (ASB) published its Financial Reporting Standard, which obliges all public companies listed on the London Stock Exchange to apply similar standards. Although some differences exist between IAS-19 and FRS-17, the bottom line remains the same; assets and liabilities should be valued on a market instead of actuarial basis.

The introduction of the above accounting standards is likely to result in more volatile financial statements than before. This will especially be the case for companies whose assets and liabilities are not well matched. This makes inflation derivatives particularly interesting for pension funds with indexation schemes. By including inflation derivatives in their investment portfolio, companies are able to match their inflation-linked liabilities more closely. This can significantly reduce the volatility of their financial statements.

## APPENDIX

### A.1. Calculating inflation asset swap spreads

*Non-accreting notional*

The asset swap seller sells the bond for par (assumed to equal 1 or 100%) plus accrued interest. The net up-front payment therefore has a value of $1\text{-}B_{MKT}$ for the par asset swap and zero for the market asset swap, where $B_{MKT}$ is the full price of the bond in the market. If we assume that both parties to the swap are AA-rated, these cashflows are discounted off the Libor curve. For simplicity we assume that all payments are annual and are made on the same dates. The breakeven asset swap spread $s$ is computed by setting the net present value of all cashflows equal to zero. From the perspective of the par asset swap seller, the present value is:

$$\underbrace{1 - B_{MKT}}_{\substack{\text{upfront payment}\\ \text{to purchase bond}\\ \text{for par}}} + \underbrace{B_{LIBOR} - (1 + s\mathrm{PV01})}_{\text{running cash flows}} = 0$$

where

$$B_{LIBOR} = c\sum_{i=1}^{N} I(0,T_i)D_n(0,T_i) + I(0,T_N)D_n(0,T_N)$$

is the value of the inflation bond priced off the Libor curve with a real coupon equal to $c$, and

$$\mathrm{PV01} = \sum_{i=1}^{N} \delta_i D_n(0,T_i)$$

is the present value of a 1bp annuity with the same schedule as the floating side of the asset swap, priced off the Libor curve. We used the fact that the value of a stream of Libor payments equals 1 and the spread results in an annuity of $s$. Solving gives us the result:

$$s = \frac{B_{LIBOR} - B_{MKT}}{\mathrm{PV01}},$$

In the case of a market asset swap, we need to solve more or less the same equation. The value for the asset swap seller is given by:

$$\underbrace{B_{MKT} - B_{MKT}}_{\substack{\text{Upfront payment to}\\ \text{purchase asset in return}\\ \text{for } B_{MKT}}} + \underbrace{B_{LIBOR} - \underbrace{B_{MKT}\left(1 + s\mathrm{PV01}\right)}_{\text{Floating payments}}}_{\text{running cash flows}} = 0$$

which results in an asset swap spread $s$ equalling:

$$s = \frac{B_{LIBOR} - B_{MKT}}{\mathrm{PV01} \times B_{MKT}}.$$

*Accreting notional*

In the case of an accreting notional, the determination of the asset swap spread is more tedious, but conceptually not much harder. We sustain the same assumptions as above, with the exception that the notionals multiplicatively increase by ($1$+$a$), where $a$ denotes the accreting factor. The value for the asset swap seller in case of a par accreting asset swap is given by:

$$\underbrace{1-B_{MKT}}_{\substack{\text{Upfront payment to}\\ \text{purchase asset in}\\ \text{return for par}}} +$$

$$\underbrace{B_{LIBOR} - \underbrace{\left(\sum_{j=1}^{N}(1+a)^j\left[D_n(0,T_{j-1})-D_n(0,T_j)\right]+s\sum_{j=1}^{N}(1+a)^j D_n^L(0,T_j)\right)}_{\text{Floating payments}}}_{\text{Running cash flows}} = 0$$

where $B_{LIBOR}$ is as before and the term between round brackets denotes the value of the accreting floating leg plus a spread. Solving for the asset swap spread, $s$, gives:

$$s = \frac{1-B_{MKT}+B_{LIBOR}-\sum_{j=1}^{N}(1+a)^j\left[D_n(0,T_{j-1})-D_n(0,T_j)\right]}{\sum_{j=1}^{N}(1+a)^j D_n(0,T_j)}$$

One can solve for the spread of an accreting market value asset swap in a similar fashion.

*Early redemption inflation asset swap*

If there is no early redemption, the value of the notional increase paid at maturity is given by:

$$V(0) = D_n(0,T_N)E_0^N\left[\frac{I(T_N)}{I(T_0)}\right] = D_n(0,T_N)E_0^N\left[\sum_{j=1}^{N}\frac{I(T_j)-I(T_{j-1})}{I(T_0)}\right] = D_n(0,T_N)\frac{I(0,T_N)}{I(0,T_0)},$$

where the superscript $N$ indicates that we take the expectation with respect to the numeraire associated with the discount bond paying at $T_N$. Now let us consider one of the early redemption payments. The payment that needs to be made at time $T_i$ is such that it is equivalent to receiving $(I(T_i)-I(T_{i-1}))/I(T_0)$ at $T_N$. We know intuitively that:

$$\begin{aligned} V_i(0) &= D_n(0,T_N)E_0^N\left[\frac{I(T_i)-I(T_{i-1})}{I(T_0)}\right] \\ &= D_n(0,T_i)E_0^i\left[\frac{I(T_i)-I(T_{i-1})}{I(T_0)}D_n(T_i,T_N)\right]. \end{aligned}$$

as the early payment is just the present value at time $T_i$ of the index increase at maturity.

## A.2. Estimation of seasonal patterns

*Seasonal dummy model*

In this section we discuss a method of computing seasonal effects. The model, which we call the *seasonal dummy model*, is given by:

$$y_t \equiv \log\left(\frac{index_t}{index_{t-1}}\right) = \sum_{i=1}^{12}\beta_i d_i + \varepsilon_t$$

where $y_{t+1}$ denotes the inflation of the index measured as the logarithm difference of the inflation index and $t$ is measured in months. The $d_i$'s, $i=1,\dots,12$ denote monthly dummies and are defined as:

$$d_i = \begin{cases} 1 & \text{for month i} \\ 0 & \text{elsewhere} \end{cases} \qquad \text{for } i=1,\dots,12,$$

where January represents month 1,…, and December represents month 12. So $d_1=1$ for January inflation, etc. Note that the $\beta_i$s do not directly represent seasonal effects, but expected inflation for a particular month. Estimation of this model is straightforward and can be performed using ordinary least squares (OLS). We subtract expected annual inflation (the average of the βs) to get the seasonal effects.

*Tramo-Seats model*

In the seasonal dummy model treated before, we did not take into account any possible mean-reversion effects in the inflation numbers. We can generalise this approach using more advanced time-series analysis. Here we describe a more advanced method, the so-called Tramo-Seats model, which is used by Eurostat to calculate its seasonally adjusted indices. The Tramo-Seats model separates inflation into four components: (1) explanatory variab1es; (2) trend; (3) seasonal; (4) irregular component. This allows us to write the inflation $y_t$ as:

$$y_t = x_t'\beta + T_t + S_t + \varepsilon_t$$

where $x_t$ denotes the explanatory regression variables known at time $t$, $T_t$ denotes the trend for period $t$, $S_t$ denotes the seasonal and $\varepsilon_t$ denotes the irregular component. The trend, seasonal and irregular components are unobserved components and modelled as ARIMA processes. In general, ARIMA models try to explain time series behaviour using autoregressive and moving average terms, and for a time series $z_t$ can be represented as:

$$z_t = \sum_{j=1}^{p} \theta_j z_{t-j} + \sum_{j=1}^{p} \Theta_j \xi_{t-j} + \xi_t$$

where the first sum denotes the autoregressive components and the second sum the moving average components. For example, we can have:

$$\begin{aligned} T_{t+1} &= T_t + \eta_t & \eta_t &\sim N(0, \sigma_\eta^2) \\ S_{t+1} &= -\sum_{j=1}^{11} S_{t+1-j} + \omega_t & \omega_t &\sim N(0, \sigma_\omega^2) \end{aligned}$$

Note that in this specification, seasonals only add up to an expected zero on an annual basis. Assuming that $\varepsilon_t$ is Gaussian distributed, this model can be estimated by maximum likelihood. See Brondolo and Giani (2005) for a more detailed analysis of seasonality in inflation indices.

## A.3. Duration and convexity analysis

Duration and convexity are the bread and butter of nominal bond traders. In this section we extend these first- and second-order risk measures to inflation securities. We recall the value of an inflation-linked bond (for convenience we have set $I(t_B)=1$) initiated at $t_B \leq 0$ and maturing at $T$:

$$I(0)B_r^c(0,T) = c\sum_{i=1}^{N}\left(\frac{1+b(0,T)+r}{1+y_n(0,T)+s}\right)^{T_i} + \left(\frac{1+b(0,T)+r}{1+y_n(0,T)+s}\right)^{T_N}$$

with $r=s=0$. We write the change in the value of an inflation-linked bond using a Taylor expansion.

$$\begin{aligned} \Delta I(0)B_r^c(0,T) \approx{}& \frac{dI(0)B_r^c(0,T)}{\partial t} + \\ &+ \left(\sum_{i=1}^{N}\frac{dI(0)B_r^c(0,T)}{\partial s_i}\Big|s_i=0\right)s_i + \left(\sum_{i=1}^{N}\frac{dI(0)B_r^c(0,T)}{\partial r_i}\Big|r_i=0\right)r_i \\ &+ \tfrac{1}{2}\left(\sum_{i=1}^{N}\frac{d^2I(0)B_r^c(0,T)}{\partial s_i^2}\Big|s_i=0\right)s_i^2 + \tfrac{1}{2}\left(\sum_{i=1}^{N}\frac{d^2I(0)B_r^c(0,T)}{\partial r_i^2}\Big|r_i=0\right)r_i^2 \\ &+ \left(\sum_{i=1}^{N}\frac{d^2I(0)B_r^c(0,T)}{\partial s_i \partial r_i}\Big|r_i=0, s_i=0\right)s_i r_i \end{aligned}$$

where $s_i$ indicates the change in $y_n(0,T_i)$ and $r_i$ the change in $b(0,T_i)$. The first term stands for the roll-down of the inflation-linked bond and is given by the change in the inflation-linked bond if the maturity decreases by a day and the nominal and inflation curves remain unchanged. The second line gives the first-order changes in the inflation-linked bond price if the nominal and/or inflation curve move. The last two lines give the second-order changes. As discussed in Section 6.2, the nominal PV01 is given by a parallel shift in the nominal term structure, that is, $s_i$=$s$.,

$$\mathrm{PV01} = \frac{\partial}{\partial s} I(0) B_r^c(0,T)\,|_{s=0} = -\sum_{i=1}^{N} cT_i \frac{I(0,T_i)D_n(0,T_i)}{1+y_n(0,T_i)} - T_N \frac{I(0,T_N)D_n(0,T_N)}{1+y_n(0,T_i)}$$

The inflation PV01 of inflation coupon bonds is given by a parallel shift in the inflation term structure, that is, $r_i$=$r$,

$$\mathrm{Inf\ PV01} = \frac{\partial}{\partial r} I(0) B_r^c(0,T)\,|_{r=0} = \sum_{i=1}^{N} cT_i \frac{I(0,T_i)D_n(0,T_i)}{1+b(0,T_i)} + T_N \frac{I(0,T_N)D_n(0,T_N)}{1+b(0,T_i)}.$$

Nominal convexity is given by:

$$\text{Nominal Convexity} = \frac{\partial^2}{\partial \mathrm{s}^2} I(0) B_r^c(0,T)\,|_{s=0} = \sum_{i=1}^{N} cT_i(T_i-1) \frac{I(0,T_i)D_n(0,T_i)}{(1+y_n(0,T_i))^2} + T_N(T_N-1)\frac{I(0,T_N)D_n(0,T_N)}{(1+y_n(0,T_N))^2},$$

and the inflation convexity is given by:

$$\text{Inflation Convexity} = \frac{\partial^2}{\partial \mathrm{r}^2} I(0) B_r^c(0,T)\,|_{r=0} = \sum_{i=1}^{N} cT_i(T_i-1) \frac{I(0,T_i)D_n(0,T_i)}{(1+b(0,T_i))^2} + T_N(T_N-1)\frac{I(0,T_N)D_n(0,T_N)}{(1+b(0,T_N))^2}.$$

Unlike one-curve products, we need to take into account the dependence between the moves in the curves for two-curve products. The cross convexity for inflation-linked bonds is given by:

$$\text{Cross convexity} = \frac{\partial^2}{\partial s \partial r} I(0) B_r^c(0,T)\,|_{s=r=0} = -\sum_{i=1}^{N} cT_i^2 \frac{I(0,T_i)D_n(0,T_i)}{(1+y_n(0,T_i))(1+b(0,T_i))} - T_N^2 \frac{I(0,T_N)D_n(0,T_N)}{(1+y_n(0,T_N))(1+b(0,T_N))}.$$

We see that the cross convexity is similar to the negative of the nominal and inflation convexity.[40] As the nominal and inflation PV01s also have opposing signs, we see that an equal move in the nominal and inflation curve only has a small impact on the value of the inflation bond. As discussed in Section 6.2, moves in the nominal and inflation curves are typically correlated. For instance, one can assume that $r_i$=$\rho_i s_i$ and apply the above techniques.

### A.4. Valuation of forward starting zero-coupon swaps

The crux of valuing forward starting inflation products is computing the expectation of the index ratio under the appropriate numeraire.

$$E_0^{T_i}\left[\frac{I(T_i,T_i)}{I(T_{i-1},T_{i-1})}\right] = \frac{I(0,T_i)}{I(0,T_{i-1})} G(0,T_i),$$

where:

[40] *It would exactly equal the negative of the nominal and inflation convexity if we had computed convexity in terms of continuously compounding rates.*

$$G(0,T_i) = \exp\left( \int_0^{T_{i-1}} \sigma_I(u,T_{i-1}) \cdot \gamma_r(u;T_{i-1},T_i) du \right)$$

denotes the forward start convexity term, and the expectation is taken under the $T_i$ - forward martingale measure. The convexity correction on period-on-period swaps is therefore a function of the volatility of the breakeven inflation rate with the maturity of the index start date, $\sigma_I(u,T_{i-1})$, the volatility of the real forward rate, $\gamma_r(u;T_{i-1},T_i)$, and their instantaneous correlation.[41] The forward starting zero-coupon swap rate is now given by:

$$b(0;T_{i-1},T_i) = \left( \frac{I(0,T_i)}{I(0,T_{i-1})} G(0,T_i) \right)^{1/(T_i - T_{i-1})} - 1. \cdot$$

Because a period-on-period inflation swap is a portfolio of forward starting inflation swaps, its fixed rate can be computed by valuing the underlying inflation legs of the forward start inflation swaps and solving for the breakeven rate on the fixed leg.

## A.5. Valuation of inflation caps and floors

In this appendix we present a valuation formula for caps and floors on the final index value. The final payoff of a caplet is given by:

$$\max\left\{ \frac{I(T_i)}{I(T_{i-1})} - 1 - K, 0 \right\} = \max\left\{ \frac{I(T_i,T_i)}{I(T_{i-1},T_{i-1})} - 1 - K, 0 \right\}.$$

Under the assumption that $I(T_i,T_i)/\mathrm{I}(T_{i-1}, T_{i-1})$ has a lognormal distribution:

$$V(0) = A_1(0)\Phi(h_+) - A_2\Phi(h_-),$$

where

$$A_1(0) = \frac{I(0,T_i)}{I(0,T_{i-1})} G(0,T_i)$$
$$A_2 \quad = 1 + K$$

and

$$h_{\pm} = \frac{\log\big(A_1(0)/A_2(0)\big) \pm \frac{1}{2}\sigma^2 T_i}{\sigma\sqrt{T_i}}.$$

In the Gaussian HJM model of Jarrow and Yildirim(2003) σ is given by:

$$\sigma = \frac{1}{T_i} \left( \int_0^{T_{i-1}} \left| \sigma(u;T_i) - \sigma(u;T_{i-1}) \right|^2 du + \int_{T_{i-1}}^{T_i} \left| \sigma(u;T_i) \right|^2 du \right)^{1/2}.$$

In the usual manner a valuation formula can be derived for floorlets. Caps and floors can then be valued as a portfolio of caplets and floorlets, respectively.

## A.6. Valuation of spread options and inflation-linked equity options

In this appendix we present a valuation formula for spread options, such as inflation spread options and the inflation-linked equity options. In general, these can be thought of as an option to exchange one asset, say X for another, say Y. Early work on these types of derivatives is presented in Margrabe (1978) and can also be found in standard textbooks, such as, Hull (2003). We consider structures with payoffs of the form:

$$\max\{Y(T) - X(T), 0\}.$$

Under the assumption that processes X and Y are lognormal distributed, with deterministic volatility functions and correlation, $\sigma_X(u)$, $\sigma_Y(u)$, and $\rho(u)$, respectively, we have the following pricing formula, known as Margrabe's formula:

[41] *Precisely, $\gamma_r(u;T_{i-1},T_i)$ denotes the volatility of $\log(D_r(t,T_{i-1})/D_r(t,T_i))$, i.e. the lognormal volatility of the forward real discount factor.*

$$V(0) = Y(0)\Phi(d_+) - X(0)\Phi(d_-),$$

where

$$d_\pm = \frac{\log\left(Y(0)/X(0)\right) \pm \frac{1}{2}\sigma_{XY}^2 T}{\sigma\sqrt{T}}$$

and

$$\sigma_{XY}^2 = \frac{1}{T}\left(\int_0^T \left|\sigma_X^2(u) + \sigma_Y^2(u) - 2\rho(u)\sigma_X(u)\sigma_Y(u)\right|^2 du\right).$$

The processes $X$ and $Y$ are given by $I_A(0)D_r(0,T)$ and $I_B(0)D_r(0,T)$, that is, inflation-linked zero-coupon bonds on index A and B, respectively. For the inflation-linked equity option, $X$ and $Y$ are given by $I(0)D_r(0,T)$ and $S(0)$, that is, an inflation-linked zero-coupon bond and the underlying equity portfolio.

## A.7. Valuation of inflation swaptions

In this appendix we present valuation formulas for fixed and floating inflation swaptions. The final payoff of a fixed receiver inflation swaption is given by the difference of the breakeven rate, $b(T;T_s,T_e)$, for an inflation swap starting at $T_s$ and maturing at $T_e$ and a pre-agreed strike, $K$, times the appropriate PV01 for the swap period, i.e.:

$$V(T) = \max\left\{(b(T;T_s,T_e) - K)\mathrm{PV01}(T;T_s,T_e), 0\right\}.$$

Under the assumption that $b(T;T_s,T_e)$ has a lognormal distribution under the measure associated with the PV01 numeraire, the value of the inflation swaption today is given by:

$$V(0) = \left[b(0;T_s,T_e)\Phi(h_+) - K\Phi(h_-)\right]\mathrm{PV01}(0;T_s,T_e),$$

where

$$h_\pm = \frac{\log\left(b(0;T_s,T_e)/K\right) \pm \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}.$$

In the same manner, the valuation formula for the fixed payer inflation swaption can be derived.

For floating inflation swaptions or real swaptions, one needs to perform a numerical integration assuming that both the nominal swap rate and the inflation swap rate are lognormally distributed. We will present the valuation formula under the assumption that both the nominal and inflation swap rates are normally distributed under the swap measure. The payoff at maturity is given by:

$$V(T) = \max\left(\left[b(T;T_s,T_e) + K - S(T;T_s,T_e)\right]\mathrm{PV01}(T;T_s,T_e)\right),$$

where $S(T;T_s,T_e)$ denotes the interest rate swap rate. Assuming that both $b(T;T_s,T_e)$ and $S(T;T_s,T_e)$ have a normal distribution under the measure associated with the PV01 numeraire, we know that the difference is also normally distributed. The value today is given by:

$$V(0) = \sigma\sqrt{T}\phi(d) + \left[b(0;T_s,T_e) + K - S(0;T_s,T_e)\right]\Phi(d),$$

where

$$d = \frac{b(0;T_s,T_e) + K - S(0;T_s,T_e)}{\sigma\sqrt{T}}$$

and

$$\sigma^2 = \frac{1}{T}\left(\int_0^T \left[\sigma_b^2(u) + \sigma_S^2(u) - 2\rho(u)\sigma_b(u)\sigma_S(u)\right]du\right),$$

where $\sigma_b(u)$ denotes the instantaneous volatility of the breakeven inflation rate, $\sigma_S(u)$ denotes the instantaneous volatility of the interest rate swap rate, and $\rho(u)$ their instantaneous correlation.

## REFERENCES

Brockwell, P. and R. Davis (1991): *Time Series: Theory and Methods*, Springer-Verlag, Berlin Heidelberg New York.

Brondolo, A. (2005): "Inflation Bonds Explained: Mastering Markets & Mechanics", Fixed Income Strategies Lehman Brothers.

Brondolo, A. and G. Giani (2005): "Seasonality in Price Indices", Fixed Income Strategies Lehman Brothers.

Campbell, J.Y. and R.J. Shiller (1996): "A Scorecard for Indexed Government Debt" in Ben S. Bernanke and Julio Rotemberg eds, *National Bureau of Economic Research Macroeconomics Annual* 1996, MIT Press: Cambridge, MA, pp. 155-197, 1996.

Deacon, M., Derry, A., and D. Mirfendereski (2004): *Inflation-indexed Securities*, 2nd edition, Wiley.

Foresi, S., Penati, A., and G. Pennacchi (1997): "Reducing the Cost of Government Debt: The Role of Inflation-linked Bonds" in M. De Cecco, L. Pecchi and G.Piga eds, *Managing Public Debt: Inflation-linked Bonds in Theory and Practice*, Cheltenham, UK, pp. 93-116.

Gong F., and E. Remolona (1996): "*Inflation Risk in the US Yield Curve: The Usefulness of Indexed Bonds*," Federal Reserve Bank of New York Research Paper No. 9637.

Hull, J. (2002): *Options, Futures and Other Derivatives*, 5th edition, Prentice-Hall.

Jarrow R. and Y. Yildirim (2003): "Pricing Treasury Inflation Protected Securities and Related Derivatives using an HJM model", *Journal of Financial and Quantitative Analysis* 38, pp. 337-358.

O'Kane, D. (2001): *Credit Derivatives Explained*, Structured Credit Research Lehman Brothers.

O'Kane, D., Naldi M, Ganapati S., Berd A., Pedersen C., Schlögl, L. and R. Mashal (2001): *The Lehman Brothers Guide to Exotic Credit Derivatives*, Structured Credit Research Lehman Brothers.

Margrabe, W. (1978): "The Value of an Option to Exchange One Asset for Another", *Journal of Finance* 33, pp. 177-186.

## GLOSSARY

**BTP€i (Buoni Poliennali del Tesoro)**

A BTP€i is an inflation-linked bond issued by Italy and linked to a European CPI index, the HICPxT.

**Bureau of Labor Statistics (BLS)**

US statistical office that is responsible for the publication of the US CPI index.

**CPI index**

Consumer price index. Both used as a general term and specifically for the US market.

**Eurostat**

European statistical office responsible for the publication of the European HICP and HICPxT indices.

**FAS 133**

The US accounting framework for all financial derivatives, which came into effect in June 2000. It requires companies to mark-to-market their derivative positions and to post gains or losses to earnings.

**FRCPI**

The French national CPI as calculated by INSEE. This CPI excludes tobacco.

**HICP / HICPxT**

Harmonized Index of Consumer Prices. HICPxT stands for HICP excluding tobacco. The HICP indices are calculated by Eurostat.

**IAS-19**

International Accounting Standard 19, the section in the new European accounting standards on employee benefits. It requires companies to recognise employee benefits at the time they are earned, not when they are payable. This requires pension funds with defined benefit schemes to mark-to-market their positions.

**IAS-39**

International Accounting Standard 39, the section in the new European accounting standards on derivative transactions. It requires companies to value derivatives on a mark to market basis.

**INSEE**

Institut National de la Statistique et des Etudes Economiques, the French national institute of statistics and economic studies.

**ISDA**

International Swaps and Derivatives Association.

**Inflation swap**

An *inflation swap* is a bilateral contract involving the exchange of inflation-linked payments for predetermined fixed or floating payments. It is typically used to hedge inflation risk. We call the inflation swap a payer inflation swap if the holder pays inflation on the swap and a receiver inflation swap if the holder receives inflation.

**Inflation asset swap**

A combination of the purchase of an inflation-linked bond and entry in a (usually) off-market floating inflation swap.

**Inflation asset swap spread**

The spread with respect to the Libor rate received by the asset swap buyer in an inflation asset swap.

**Inflation cap**

An option that provides a cashflow equal to the difference between inflation and a pre-agreed strike rate if this difference is positive.

**Inflation floor**

An option that provides a cashflow equal to the difference between a pre-agreed strike rate and inflation if this difference is positive.

**Inflation future**

Inflation futures are futures on the inflation indices (US CPI and HICPxT) and are traded at the Chicago Mercantile Exchange (CME).

**Inflation payer**

An inflation payer is a party which acts as a payer in the inflation market. Typically, inflation payers have an income stream linked to an inflation index. The typical example is a sovereign whose tax revenues grow with rising inflation. Paying inflation in the inflation market allows them to smooth their real income.

**Inflation receiver**

An inflation receiver is a party which acts as a receiver in the inflation market. Typically, inflation receivers have a liability stream linked to an inflation index. The typical example is a pension fund whose pensions are linked to an inflation index. Receiving inflation in the inflation market provides them with a natural hedge against their inflation risk.

**Inflation swaption**

An option to enter into an inflation swap (either zero-coupon or period-on-period). The swaption is denoted a payer swaption when it gives the right to enter into a payer inflation swap and a receiver swaption when it gives the right to enter into a receiver swaption.

**Interest rate swap**

A bilateral derivative contract involving the exchange of fixed rate payments for floating rate payments typically linked to a particular Libor rate. Typically used to hedge interest rate risk.

**Libor**

The London Inter-Bank Offered Rate. This is an interest rate at which highly rated (typically AA-rated) banks can borrow. It is calculated by polling 16 banks on a daily basis (through their London branches) to determine the rate at which they can borrow for various terms and currencies. For each term and currency the received rates are ranked, the top and bottom four are deleted, and an average of the remaining eight is taken.

**LPI**

Limited Price Index.

**National statistics**

The UK statistical office responsible for the publication of the RPI and RPIX indices.

**OAT€i/i (Obligations assimilables du Trésor)**

An OATi is an inflation-linked bond issued by France and linked to a French CPI index, FRCPI. An OAT€i is an inflation-linked bond issued by France and linked to a European CPI index, the HICPxT.

**PV01**

Present value of one basis point. Also referred to as PVBP and annuity.

## SUMMARY OF NOTATION AND DEFINITIONS

The following notation is used in the document.

In general we use sub-indices of *n* to indicate nominal and sub-indices of *r* to indicate real.

$\boldsymbol{D_n(0,T)}$

denotes the value at time *t* of a *nominal discount bond* with a notional of 1 nominal unit (eg, euro) maturing at time *T* in nominal units (eg, euros).

$\boldsymbol{D_r(0,T)}$

denotes the value at time *t* of a *real discount bond* with a notional of 1 real unit maturing at time *T* in real units.

$\boldsymbol{I(t)}$

denotes the (*daily*) *reference number* at time *t*. The daily reference number is linked to an inflation index (for example, the HICPxT index). It gives the number of nominal units per inflation index unit, and can be seen as an exchange rate from the nominal to the real economy ($I(t)$ units of nominal value correspond to 1 unit of real value). $I(t)$ is usually lagged a few months. For instance, the daily reference number at 1 April 2004 denotes the January 2004 CPI level in the euro market.[42] Note the difference between inflation index values and daily reference numbers. *Inflation index values* are denoted by eg, *HICPxT*(*Jan04*) and only exist for whole months, whereas *daily reference numbers* exist for each day and are denoted by eg, *I*(*15/01/04*).

$\boldsymbol{i(0,T)}$

denotes the *annualised inflation* between 0 and *T*. i(0,*T*) is related to the inflation daily reference numbers by:

$$i(0,T) = \left(\frac{I(T)}{I(0)} - 1\right)^{1/T} - 1,$$

where *T* is measured in years.

$\boldsymbol{y_k(0,T)}$**, k** $\in$ **{n,r}**

denotes the (annual compounding) *zero-yield* on a discount bond and is defined by:

$$y_k(0,T) = D_k(0,T)^{-1/T} - 1.$$

$\boldsymbol{B_n^c(0,T)}$

denotes the time t value (dirty price) of a *coupon-bearing bond* in the nominal economy with maturity $T=T_N$ and face value 1, that is:

$$B_n^c(0,T) = \sum_{i=1}^{N} cD_n(0,T_i) + D_n(0,T_N)$$

$\boldsymbol{B_r^c(0,T)}$

denotes the time t value of a *real coupon-bearing bond* (the coupon is sometimes referred to as the real-rate coupon) in the real economy with maturity $T=T_N$ and face value 1, that is:

$$B_r^c(0,T) = \sum_{i=1}^{N} cD_r(0,T_i) + D_r(0,T_N)$$

$\boldsymbol{y_k^c(0,T)}$**, k** $\in$ **{n,r}**

denotes the (annual compounding) *yield to maturity* on a bond and is (implicitly) defined by:

$$B_k^c(0,T) = \sum_{i=1}^{N} \frac{c}{(1+y_k^c(0,T))^{T_i}} + \frac{1}{(1+y_k^c(0,T))^{T_N}}.$$

[42] *For notational convenience, we will in general denote the inflation index by CPI. Depending on the market we discuss this could mean HICPxT, RPI, and so on.*

The views expressed in this report accurately reflect the personal views of Jeroen Kerkhof, the primary analyst(s) responsible for this report, about the subject securities or issuers referred to herein, and no part of such analyst(s)' compensation was, is or will be directly or indirectly related to the specific recommendations or views expressed herein.

Any reports referenced herein published after 14 April 2003 have been certified in accordance with Regulation AC. To obtain copies of these reports and their certifications, please contact Larry Pindyck (lpindyck@lehman.com; 212-526-6268) or Valerie Monchi (vmonchi@lehman.com; 44-(0)207-102-8035).

Lehman Brothers Inc. and any affiliate may have a position in the instruments or the Company discussed in this report. The Firm's interests may conflict with the interests of an investor in those instruments.

The research analysts responsible for preparing this report receive compensation based upon various factors, including, among other things, the quality of their work, firm revenues, including trading, competitive factors and client feedback.



LEHMAN BROTHERS

*Where vision gets built.®*